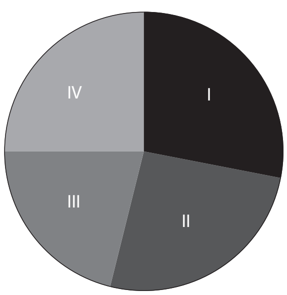
Middle School Mathematics (0069)

Test at a Glance

Test Name	**Middle School Mathematics**
Test Code	**0069**
Time	**2 hours**
Number of Questions	**45 Multiple-choice (Part A)** **3 Short constructed-response (Part B)**
Format	**Multiple-choice questions and constructed-response questions; graphing calculator allowed; calculators with QWERTY keyboards not allowed**
Weighting	**Multiple-choice: 75% of total score** **Short constructed-response: 25% of total score**



	Content Categories	Number of Questions	Percentage of Total Score
I.	Arithmetic and Basic Algebra	16-18	26-30%
II.	Geometry and Measurement; Coordinate Geometry, Functions and Graphs	14-18	22-30%
III.	Data, Probability, Statistical Concepts; Discrete Mathematics, and Computer Science	11-16	18-26%
IV.	Problem-Solving Exercises (including Content-Specific Pedagogy)	3 (constructed-response)	25%

Pacing and Special Tips	**In allocating time on this assessment, it is expected that about 90 minutes will be spent on the multiple-choice section and about 30 minutes will be spent on the constructed-response section; the sections are not independently timed.**

About this test

Middle School Mathematics is designed to certify examinees as teachers of middle school mathematics. Examinees have typically completed a bachelor's program with an emphasis in mathematics education, mathematics, or education. Coursework will have included many of the following topics: theory of arithmetic, foundations of mathematics, geometry for elementary and middle school teachers, algebra for elementary and middle school teachers, the big ideas of calculus, data and its uses, elementary discrete mathematics, elementary probability and statistics, history of mathematics, mathematics appreciation, and basics of computers.

The examinee will be required to work with mathematical concepts, reason mathematically, make conjectures, see patterns, justify statements using informal logical arguments, and construct simple proofs. Additionally, the examinee will be expected to solve problems by integrating knowledge from different areas of mathematics, to use various representations of concepts, to solve problems that have several solution paths, and to develop mathematical models and use them to solve real-world problems.

The examinee will be allowed to use a scientific or graphing calculator during the examination; however, calculators with QWERTY keyboards will not be allowed.

The test is designed to conform with the recommendations in the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics* (1989), *Professional Standards for Teaching Mathematics* (1991), and *Assessment Standards for School Mathematics* (1995.)

Topics Covered

In each of the content categories, the test will assess an examinee's ability to use appropriate mathematical language and representations of mathematical concepts, connect mathematical concepts to one another and to real-world situations, and integrate mathematical concepts to solve problems.

Because the assessments were designed to measure the ability to integrate knowledge of mathematics, answering any question may involve more than one competency and may involve competencies from more than one content category.

Representative descriptions of topics covered in each category are provided below.

I. Arithmetic and Basic Algebra

- Add, subtract, multiply, and divide rational numbers expressed in various forms; apply the order of operations; identify the properties of the basic operations on the standard number systems (for example, closure, commutativity, associativity, distributivity); identify an inverse, the additive and multiplicative inverses of a number; use numbers in a way that is most appropriate in the context of a problem
- Order any finite set of real numbers and recognize equivalent forms of a number; classify a number as rational, irrational, real, and/or complex; estimate values of expressions involving decimals, exponents, and radicals; find powers and roots
- Given newly defined operations on a number system, determine whether the closure, commutative, associative, or distributive properties hold
- Demonstrate an understanding of concepts associated with counting numbers (for example, prime or composite, even or odd, factors, multiples, divisibility)
- Interpret and apply the concepts of ratio, proportion, percent in appropriate situations
- Recognize the reasonableness of results within the context of a given problem; using estimation, test the reasonableness of results
- Work with algebraic expressions, formulas, and equations; add, subtract, and multiply polynomials; divide polynomials; add, subtract, multiply, and divide algebraic fractions; perform standard algebraic operations involving complex numbers, radicals, and exponents, including fractional and negative exponents
- Solve and graph linear equations and inequalities in one or two variables; solve and graph systems of linear equations and inequalities in two variables; solve and graph nonlinear algebraic equations; solve equations and inequalities involving absolute values

Middle School Mathematics (0069)

IIa. Geometry and Measurement

- Solve problems that involve measurement in both the metric and traditional systems (stress is on estimation and problems with context; provided are conversion factors when appropriate and formulas for all but the most basic measurement tasks)
- Compute perimeter and area of triangles, quadrilaterals, and circles and regions that are combinations of these figures; compute the surface area and volume of right prisms, cones, cylinders, and spheres and solids that are combinations of these figures
- Apply the Pythagorean Theorem to solve problems; solve problems involving special triangles, such as isosceles and equilateral
- Use relationships such as congruency and similarity to solve problems involving two-dimensional and three-dimensional figures; solve problems involving parallel and perpendicular lines
- Solve problems using the relationships among the parts of triangles, such as sides, angles, medians, midpoints, and altitudes
- Solve problems using the properties of special quadrilaterals, such as the square, rectangle, parallelogram, rhombus, and trapezoid; describe relationships among sets of special quadrilaterals; solve problems involving angles, diagonals, vertices of polygons with more than four sides
- Solve problems that involve using the properties of circles, including those involving inscribed angles, central angles, chords, radii, tangents, secants, arcs, and sectors
- Solve problems involving reflections, rotations, and translations of points, lines, or polygons in the plane
- Execute geometric constructions using straight-edge and compass (for example, bisect an angle or construct a perpendicular); prove that a given geometric construction yields the desired result
- Estimate actual and relative error in the numerical answer to a problem by analyzing the effects of roundoff and truncation errors introduced in the course of solving a problem
- Demonstrate an intuitive understanding of a limit
- Demonstrate an intuitive understanding of maximum and minimum
- Estimate the area of a region in the xy-plane

IIb. Coordinate Geometry, Functions and Their Graphs

- Understand function notation for functions of one variable and be able to work with the algebraic definition of a function (that is, for every x there is at most one y)
- Identify whether a graph in the plane is the graph of a function; given a set of conditions, decide if they determine a function
- Given a graph, for example, a line, a parabola, a step, absolute value, power of 2, or simple exponential, select an equation that best represents the graph; given an equation, show an understanding of the relationship between the equation and its graph
- Determine the graphical properties and sketch a graph of a linear, step, absolute value, or quadratic function
- Demonstrate an understanding of a physical situation or a verbal description of a situation and develop a model of it such as a chart, graph, equation, story, or table
- Determine whether a particular mathematical model, such as an equation, can be used to describe two seemingly different situations. For example, given two different word problems, determine whether a particular equation can represent the relationship between the variables in the problems
- Find the domain (x-values) and range (y-values) of a function (without necessarily knowing the definitions); recognize certain properties of graphs (for example, slope, intercepts, intervals of increase or decrease, axis of symmetry)

- Determine the equations of lines given sufficient information; recognize and use the basic forms of a straight line
- Solve problems that can be represented on the *xy*-plane (for example, finding the distance between two points, or finding the coordinates of the midpoint of a line segment)
- Translate verbal expressions and relationships into algebraic expressions or equations; provide and interpret geometric representations of numeric and algebraic concepts
- Solve problems that involve quadratic equations using a variety of methods (graphing, formula, calculator)

III a. Data, Probability, and Statistical Concepts

- Organize data into a presentation that is appropriate for solving a problem (for example, construct a histogram and use it in the calculation of probabilities)
- Read and analyze data presented in various forms (i.e., tables, charts, graphs, line, bar, circle, double line, double bar, scatterplot, stem-and-leaf, line plot, box and whiskers); draw conclusions from data
- Solve probability problems involving finite sample spaces by actually counting outcomes; solve probability problems using counting techniques; solve probability problems involving independent and dependent trials; solve problems using geometric probability; solve problems that can have only two outcomes (for example, male or female, heads or tails, prime or not prime)
- Solve problems involving average, including arithmetic mean and weighted average; find and interpret common measures of central tendency (mean, sample mean, median, mode) and know which is the most meaningful to use in a given situation; find and interpret common measures of dispersion (for example, range, spread of data, outliers)
- Work with a probability distribution at an intuitive level (for example, solve a problem involving all possible outcomes from tossing a pair of numbered cubes)

III b. Discrete Mathematics and Computer Science

- Use and interpret statements that contain logical connectives (and, or, if—then) as well as logical quantifiers (some, all, none)
- Solve problems involving the union and intersection of sets, subsets, and disjoint sets
- Solve basic counting problems involving permutations and combinations without necessarily knowing formulas; use Pascal's triangle to solve problems
- Work with numbers that can be expressed in bases other than 10
- Solve problems that involve simple sequences or number patterns (for example, triangular numbers or other geometric numbers); find rules for number patterns
- Use and interpret simple matrices as a tool for displaying data
- Draw conclusions from information contained in simple diagrams, flowcharts, paths, circuits, networks, or algorithms
- Use the calculator as a tool to explore patterns, make conjectures, make predictions, make generalizations; know when to use a calculator
- Demonstrate an understanding of basic computer terminology and the roles of hardware and software; use "friendly" software (for example, spreadsheets, instruction packages)

Middle School Mathematics (0069)

IV. Problem-Solving Exercises (including Content-Specific Pedagogy)

Part B of the test contains three equally weighted constructed-response questions that together comprise 25% of the examinee's score. Each of the three previously described content categories (I, II and III) will be the primary focus of one of the three constructed-response questions. The examinee may also be expected to integrate knowledge from different areas of mathematics. The following content-specific pedagogy topics may be included in any or all of the three constructed-response questions.

- Recognize errors in student work and the underlying misconceptions; suggest ways to help a student develop correct concepts
- Identify the prerequisite knowledge and skills students might need to possess in order to correctly learn a particular topic
- Develop questions that might be asked orally that will best assess a student's conceptual understanding of a particular topic
- Given a particular problem, identify several problem-solving strategies that can be used to explore or solve the problem with students (for example, guess and check, reduce to a simpler problem, draw a diagram, work backwards)
- Use representations of mathematical concepts (for example, analogies, drawings, examples, symbols, manipulatives) that have the potential to help students understand and learn mathematical concepts
- Use a variety of teaching strategies (for example, laboratory work, supervised practice, group work, lecture) appropriate for a particular topic or unit and also for a particular group of students
- Integrate concepts to show relationships among topics
- Relate mathematical concepts and ideas to real-world situations
- Identify, evaluate, and use curricular materials and resources for mathematics instructions in ways appropriate for a particular group of students and a particular topic; know when to use technology, particularly calculators
- Identify, evaluate, and use appropriate evaluation strategies (for example, observations, interviews, questioning, oral discussions, written tests, portfolios) to assess student progress in mathematics; write questions that test specific mathematical skills; develop a set of questions that can be used to probe for both procedural and conceptional understanding
- Determine appropriate strategies to solve a given problem. Strategies might include conjectures, counterexamples, inductive reasoning, deductive reasoning, proof by contradiction, direct proof, and other types of proof, using tools (i.e., mental math, pencil and paper, calculator, computer, models, trees and graphs)
- Demonstrate with examples when each of the strategies listed above would be appropriate to use.
- After solving a problem, reflect on the strategies used; consider if there are other appropriate strategies; if there are more appropriate strategies; if the strategies employed can be used to solve other types of problems; if the strategies can be used to solve a more general class of problems
- Communicate the results of reasoning in an appropriate form (for example, the written word, tables, charts, and graphs); explain the processes used in solving a problem

The sample questions that follow illustrate the types of questions in the test. They are not, however, representative of the entire scope of the test in either content or difficulty. Answers with explanations follow the questions.

Directions: Each of the questions or incomplete statements below is followed by four suggested answers or completions. Select the one that is best in each case.

x	y
-4	-2
-3	$-\frac{3}{2}$
-2	-1
-1	$-\frac{1}{2}$
0	0

1. Which of the following is true about the data in the table above?

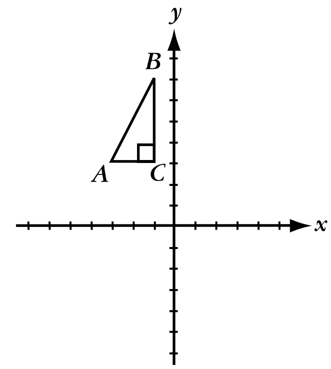
   (A) As x decreases, y increases.
   (B) As x increases, y does not change.
   (C) As x increases, y decreases.
   (D) As x increases, y increases.

2. The average number of passengers who use a certain airport each year is 350 thousand. A newspaper reported the number as 350 million. The number reported in the newspaper was how many times the actual number?

   (A) 10
   (B) 100
   (C) 1,000
   (D) 10,000

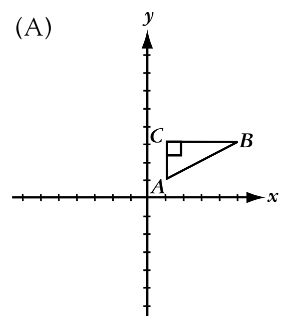
3. If there are exactly 5 times as many children as adults at a show, which of the following CANNOT be the number of people at the show?

   (A) 102
   (B) 80
   (C) 36
   (D) 30

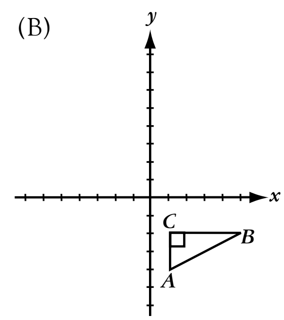


4. Which figure below results if right triangle ABC above is flipped (reflected) across the y-axis and then turned (rotated) clockwise about point C by 90 degrees?

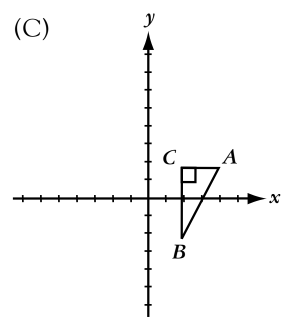
(A)



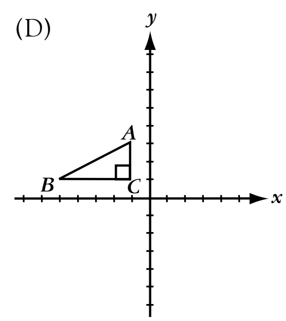
(B)

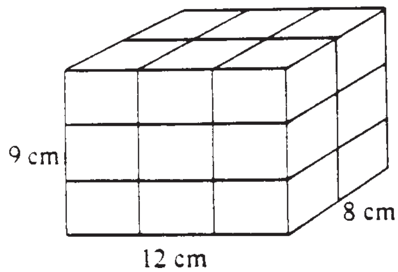


(C)



(D)



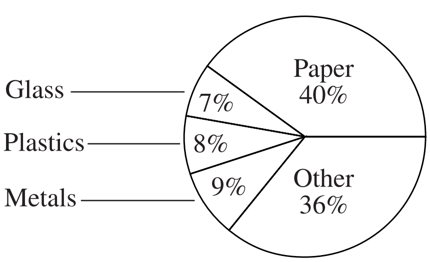


5. The large rectangular block pictured above was made by stacking smaller blocks, all of which are the same size. What are the dimensions in centimeters of each of the smaller blocks?

   (A) $3 \times 2 \times 3$
   (B) $3 \times 3 \times 3$
   (C) $3 \times 4 \times 3$
   (D) $4 \times 4 \times 3$

6. In a class of 29 children, each of 20 children has a dog and each of 15 has a cat. How many of the children have both a dog and a cat?

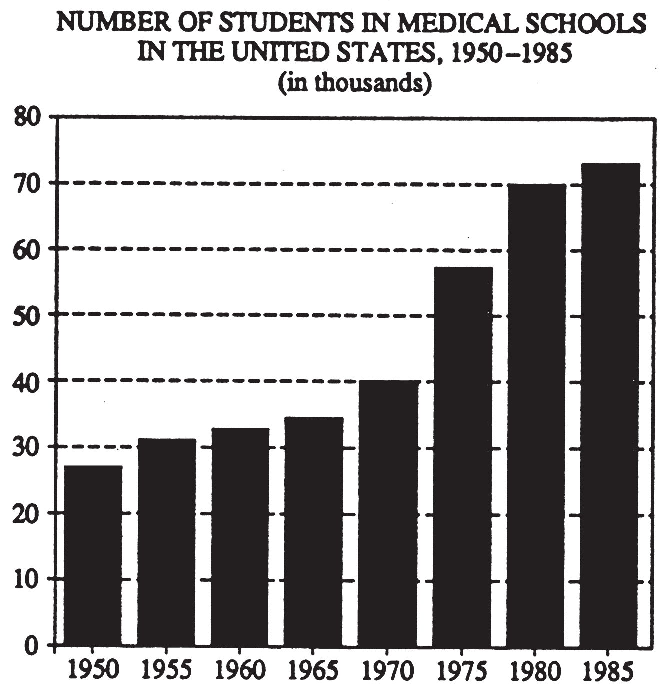
   (A) None of the children necessarily has both.
   (B) Exactly 5
   (C) Exactly 6
   (D) At least 6 and at most 15



7. The graph above shows the distribution of the content, by weight, of a county's trash. If approximately 60 tons of the trash consists of paper, approximately how many tons of the trash consists of plastics?

   (A) 24
   (B) 20
   (C) 15
   (D) 12

Questions 8-9 refer to the following graph.



8. In how many of the years shown were there more than twice as many students in medical schools as there were in 1950?

   (A) None
   (B) One
   (C) Two
   (D) Three

9. The number of students in medical schools increased by approximately what percent from 1970 to 1980?

   (A) 75%
   (B) 60%
   (C) 50%
   (D) 45%

10. In order to estimate the population of snails in a certain woodland, a biologist captured and marked 84 snails that were then released back into the woodland. Fifteen days later the biologist captured 90 snails from the woodland, 12 of which bore the markings of the previously captured snails.

    If all of the marked snails were still active in the woodland when the second group of snails were captured, what should the biologist estimate the snail population to be based on the probabilities suggested by this experiment?

    (A) 630
    (B) 1,010
    (C) 1,040
    (D) 1,080

11. If a student takes a test consisting of 20 true-false questions and randomly guesses at all of the answers, what is the probability that all 20 guesses will be correct?

    (A) 0

    (B) $\left(\frac{1}{2}\right)^{20}$

    (C) $\frac{1}{2(20)}$

    (D) $\frac{1}{2}$

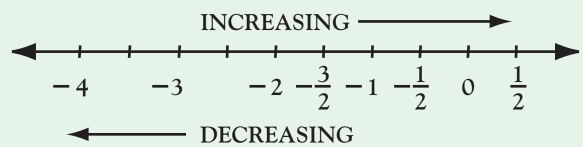
ROBIN'S TEST SCORES
88, 86, 98, 92, 90, 86

12. In an ordered set of numbers, the median is the middle number if there is a middle number; otherwise, the median is the average of the two middle numbers. If Robin had the test scores given in the table above, what was her median score?

    (A) 89
    (B) 90
    (C) 92
    (D) 95

Answers

1. As x moves from -4 to 0, that is, from left to right on the number line, its value increases.



Similarly, the value of y increases from -2 to 0. Thus, it can be seen that as x increases, y increases. The answer is D.

2. The number of passengers who use the airport each year, 350 thousand, can be written as 350,000; 350 million can be written as 350,000,000. 350,000,000 = 1,000 × 350,000, so the correct answer is C.

3. If a represents the number of adults, then $5a$ represents the number of children and $6a$ represents the total number of people at the show. Since $6a$ represents a whole number that is a multiple of 6, there cannot be 80 people at the show, for 80 is not a multiple of 6. The correct answer is B.

4. When triangle ABC is reflected across the y axis, the figure formed is located in quadrant I and is the mirror image of the given figure. Rotating the triangle about vertex C by 90 degrees yields choice A.

5. The length of the large block, 12 centimeters, is 3 times the length of a small block, so each small block is 12 ÷ 3 = 4 centimeters long. Similarly, the width of a small block is 8 ÷ 2 = 4 centimeters, and the height of a small block is 9 ÷ 3 = 3 centimeters. Thus, the correct answer is D.

6. Since the 29 children have a total of 35 dogs and cats, at least 6 children must have both a dog and a cat. If there are exactly 6 children with both a cat and a dog, then 14 children have only a dog and 9 children have only a cat. On the other hand, all 15 cat owners could also own a dog; then 5 children have only a dog and 9 children have neither a dog nor a cat. Thus, the correct answer is D.

7. The circle graph shows the distribution of the trash content *in percent;* the question asks for the *weight* of the plastics content in tons. From the graph we see that plastics account for 8% of the total weight of the trash. The problem states that 60 tons of the trash consist of paper; the graph shows that this amount equals 40% of the total, so

$60 = 0.4 \times \text{(total weight)}$

and the total weight is $\frac{60}{0.4} = 150$ tons.

The weight of plastics equals 8% of 150 tons or $(0.08)(150) = 12$ tons.

There is another, slightly faster, way to solve this problem. We use the fact that the *ratio* of plastics to paper in the trash is the same, whether the two amounts are given as percents or in tons. This gives us the proportion

$$\frac{\text{tons of plastic}}{\text{tons of paper}} = \frac{8\%}{40\%} = \frac{1}{5}$$

or

$$\frac{\text{tons of plastic}}{60} = \frac{1}{5}$$

$$\text{tons of plastic} = \frac{60}{5} = 12$$

The answer is D.

8. There is information for eight different years in the bar graph. The vertical scale goes from 0 to 80,000. The zeros are left off the scale because the title tells you to read the numbers in thousands. To find the number of students in any one year, read the height of the corresponding bar off the left-hand scale and multiply that height by 1,000.

The bar for 1950 has a height of about 27, so the number of students in 1950 was about 27,000. You have to find the number of years in which there were more than twice as many, that is, more than 54,000 students. To do this, count the number of bars that are higher than 54. These are the bars for 1975, 1980, and 1985. Thus, there were three years in which there were more than twice as many students as in 1950. The answer is D.

9. To compute a percentage increase, you need the increase in the number of students and the number of students before the increase.

The graph shows that the number of students in 1970 was 40,000 and the number of students in 1980 was 70,000, an increase of 30,000 students. To find the percent increase, divide this number by the base number, that is, the number of students before the increase, or 40,000.

$$\frac{30,000}{40,000} = \frac{3}{4} = 0.75 = 75\%$$

The answer is A.

10. Given the conditions of the experiment it is reasonable to assume that the 90 snails captured by the biologist 15 days after the markings were made were a random sample of all the snails.

Thus, about $\frac{12}{90}$, or $\frac{2}{15}$, of the population had been marked. Thus, the original 84 snails marked represented approximately $\frac{2}{15}$ of the entire population and the biologist should estimate the snail population to be $84 \times \frac{15}{2}$, or 630.

The correct answer is A.

11. The probability that the student guesses any one answer correctly is 1/2, and, since the student is randomly guessing, the guesses are independent events. Thus, the probability of guessing all 20 answers correctly is $\left(\frac{1}{2}\right)^{20}$, and the correct answer is B.

12. The problem gives a set of test scores and the definition of median. The first part of the definition tells you to order the scores, that is, to arrange them in order from smallest to largest. Here are the numbers ordered from smallest to largest:

86, 86, 88, 90, 92, 98

Because there is an even number of scores (6), there are two middle numbers in the set, 88 and 90, and the average of the two middle numbers is

$$\frac{88 + 90}{2} = \frac{178}{2} = 89$$

Thus the median of Robin's scores is 89 and the answer is A. (Notice that the median of a set of numbers need not be one of the numbers in the set.)

This section presents sample questions and constructed-response samples along with the standards used in scoring the responses. When you read these sample responses, keep in mind that they will be less polished than if they had been developed at home, edited, and carefully presented. Examinees do not know what questions will be asked and must decide, on the spot, how to respond. Readers take these circumstances into account when scoring the responses.

Readers will assign scores based on the following scoring guide.

SCORING GUIDE

3

- Responds appropriately to all parts of the question
- Where required, provides a strong explanation that is well supported by relevant evidence
- Demonstrates a strong knowledge of subject matter, concepts, theories, facts, procedures, or methodologies relevant to the question
- Demonstrates a thorough understanding of the most significant aspects of any stimulus material presented

2

- Responds appropriately to most parts of the question
- Where required, provides an explanation that is sufficiently supported by relevant evidence
- Demonstrates a sufficient knowledge of subject matter, concepts, theories, facts, procedures, or methodologies relevant to the question
- Demonstrates a basic understanding of the most significant aspects of any stimulus material presented

1

- Responds appropriately to some part of the question
- Where required, provides a weak explanation that is not well supported by relevant evidence
- Demonstrates a weak knowledge of subject matter, concepts, theories, facts, procedures, or methodologies relevant to the question
- Demonstrates little understanding of significant aspects of any stimulus material presented

0

- Blank, off-topic, or totally incorrect response
- Does nothing more than restate the question or some phrases from the question
- Demonstrates very limited understanding of the topic

Sample Question: Data Interpretation

The heights, in inches, of the 17 people in a certain club are listed below.
49, 54, 54, 55, 56, 57, 60, 61, 61, 64, 64, 64, 65, 66, 67, 68, 72

A. Define and identify each of the following for the list above: median and range.
B. Define and compute the arithmetic mean for the list above.
C. Draw a stem-and-leaf plot of the data using the tens digits as the stems and the units digits as the leaves.

Sample Response That Received a Score of 3:

A. The median is the middle number when the numbers are arranged by size. The median height is 61. The range is the difference between the highest and lowest score. The range for this set of data is 72 − 49 = 23.
B. The arithmetic mean is the sum of the entries divided by the number of entries. 1037/17 = 61. The mean is 61.
C.

Stem	Leaf
4	9
5	4, 4, 5, 6, 7
6	0, 1, 1, 4, 4, 4, 5, 6, 7, 8

Sample Response That Received a Score of 1:

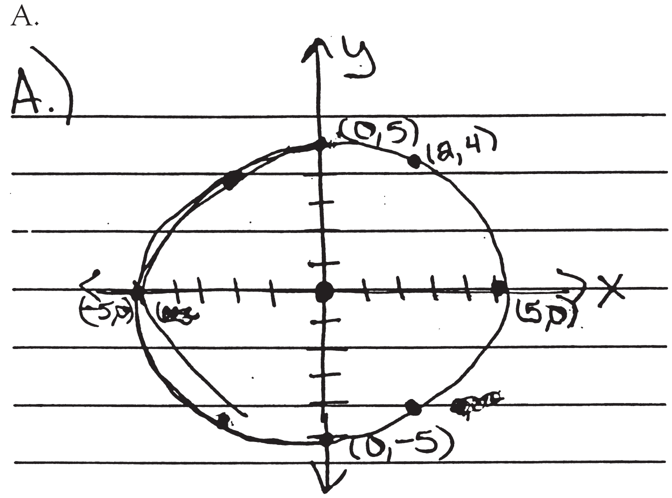
A. Median is the middle # listed. It can be found by listing the numbers ascending order & finding the # in the middle 61. The range of the numbers is 49 to 72.
B. The arithmetic mean is the average of the heights.

Sample Question: Geometry and Basic Algebra

A. On the *xy*-plane, graph the circle $x^2 + y^2 = 25$ with center at (0, 0).
B. Indicate on the circle you have graphed approximately where the points are that have a *y*-coordinate that is twice its *x*-coordinate. Explain your reasoning, perhaps by demonstrating geometrically why these points would in fact, satisfy the condition.
C. Find the coordinates of the points you have indicated in B. Explain your process and reasoning.

Sample Response That Received a Score of 2:

A.

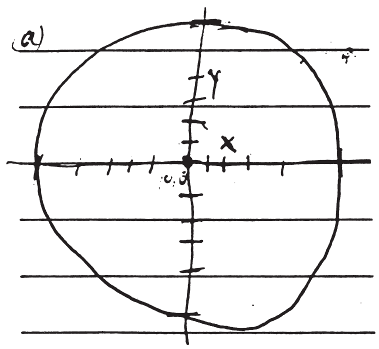


Center (0, 0)   Radius 5

B. I basically looked at points on the circle that would satisfy the condition $y = 2x$.
C. (2, 4), (2, −4), (−2, −4), (−2, 4) The technique I used was find a *y*-coordinate on the circle and then find an *x*-coordinate that was half of the *y*-coordinate. The above coordinates fit the condition, however some coordinates have different signs. The coordinates are all equal distance from the center. By using algebra,
$x^2 + (2x)^2 = 25,\ 5x^2 = 25,\ x^2 = 5,\ x = \pm\sqrt{5},\ y = \pm 2\sqrt{5}.$

Sample Response That Received a Score of 1:

A. Center (0, 0)
radius = 5
$x^2 + y^2 = 25$



B. If $y = 4$, then $x = 2$.
$x + y = 5$