School of Graduate Studies
Bemidji State University
1500 Birchmont Dr NE, #27
Bemidji, MN 56601-2699
218-755-2027

HOW TO REACH ALL TYPES OF LEARNERS THROUGH THE USE OF MANIPULATIVES IN GRADES THREE, FOUR, AND FIVE

by

Cadie Fairchild

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THIS RESEARCH PAPER HAS BEEN APPROVED ON THE DATE SHOWN BELOW:

______________________________________ ______________________

Dr. Todd Frauenholtz, Date
Committee Chair
Professor, Department of Mathematics and Computer Science

______________________________________ ______________________

Dean, School of Graduate Studies Date

HOW TO REACH ALL TYPES OF LEARNERS THROUGH THE USE OF MANIPULATIVES IN GRADES THREE, FOUR, AND FIVE

Cadie Fairchild

This paper is an examination of research concerning the use of manipulatives with third, fourth, and fifth grade mathematics instruction. The research discusses the power of child discovery, stages of representation, using manipulatives to support learning, eliminating manipulatives prematurely, and eliminating negative stereotypes linked to manipulative use.

A major concern of teaching with manipulatives is the knowledge of the teacher. An educator's role is to provide the best learning opportunities for students. Research reviewed for this paper indicates frequent hands on activities using manipulatives. They need to be aware of the many learning stages and how to address each individual learner to optimize the student's learning potential. They need to be prepared and organized while teaching with learning tools to prevent them from becoming a distraction. Professional development is crucial for teachers, advocacy of manipulatives, and diminishing negativity surrounding learning tools.

Research provided gave examples of how manipulatives can be beneficial to the classroom. Students at the concrete stage can benefit from hands on learning tools that continue to support them during educational growth and transitioning to higher cognitive thinking. Physical and virtual manipulatives help motivate students while engaging them during mathematics lessons.

Approved by:

Committee Chair	Date
Committee Member	
Committee Member	
Graduate Faculty Representative	

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Chapter 1: Introduction

I loved elementary school. I thrived in the primary grades and was pleased with the work I accomplished, which my mother would proudly hang on our refrigerator for the world to see. I understood both reading and mathematics and felt confident in the subject areas. My report card was full of E's (Excellent) in each of the subject areas.

As I progressed into the upper grades of elementary school, my report card began showing struggles in mathematics. My E's quickly turned to G's (Good) and eventually to S's (Satisfactory). I did not like to think of myself as a satisfactory learner, and this upset me. Math became uncomfortable for me. I began to believe mathematics was not my strong suit and I evolved to the point where I did not like mathematics. In fact, I went as far as saying I hated mathematics. I would often describe myself as a hands-on learner, not a "math brain." This lasted well into high school and college.

It was not until I took a course demonstrating how to teach children mathematics that my attitude regarding mathematics changed. My frustration quickly turned to excitement as I realized that this was the mathematics I remembered and in which I thrived. I understood math again. The difference: Manipulatives! Visuals and physical learning tools was the key to my understanding and my success as a student of mathematics.

Significance of the Research Problem

Tools to support learning have been around since before the one room school house. Physical objects have been used by many ancient civilizations to help solve daily problems. The Incas used knotted strings they called *Quipu,* as a counting tool. Southwest Asia civilizations used trays covered with sand that they would draw tally marks into to help them track their inventory. Counting devises that used corn kernels strung and stretched across a wooden frame were used by the Mayans and the Aztecs as a means of counting. Later, the ancient Romans created the world's first abacus by modifying the counting boards (Cole, 1961). Modern manipulatives were first designed in the late 1800s for teaching mathematical concepts. Educator Maria Montessori emphasized the importance of using manipualtives in education in the early 1900s. Many materials were designed by her to help elementary students learn fundamental concepts in mathematics (Cole, 1961). Montessori explains "[w]e cannot create observers by saying, "observe," but by giving them the power and the means for this observation, and these means are procured through education of the senses" (2004, p. 184).

The National Council of Teachers of Mathematics (NCTM) has encouraged the use of manipulatives for many years in various publications such as Teaching Children Mathematics as well as NCTM's Principles and Standards for School Mathematics. NCTM suggests using manipulatives in all grade levels to teach fundamental mathematical concepts (NCTM, 2000). Manipulatives have been introduced to students as visual aids and hands on tools to help children grasp a concept being taught. Primary education relies on manipulatives to foster early learning and engage concrete learners. All students learn differently and teachers need to teach according to each student's individualized style of learning. "Students exhibit different talents, abilities, achievements, needs, and interests in mathematics" (National Council of Teachers of Mathematics [NCTM], 2000, p. 5). So why do we see such a drastic decrease of learning tools in the classroom as students enter the upper grades of elementary school?

Statement of the Problem

The problem is that often times teachers see the importance of manipulatives in upper elementary grades three, four, and five diminish as several students begin to enter the pictorial and abstract representation stages. The reasoning behind the author's interest in classroom tools for learning is her belief there is an imprudent urgency to move students away from the concrete stage to abstract learning. Some students are ready for abstract learning after understanding concrete and pictorial representations, but many students remain tactile learners well into high school and college (Epstein, 1998). The author believes the concrete stage can continue to support students well into the pictorial representational stage of mathematical modeling. Many teachers have difficulty supporting their tactile learners because they are abstract learners themselves and cannot relate to the concrete operational stage. Freer Weiss states, "Teachers who do not have the mathematical competence to use manipulatives to convert ideas into concrete experiences will not be effective" (2006, p. 241).

Research Questions

1. What is the importance of manipulatives?
    - Why use manipulatives and how can manipulatives help a child grasp mathematical concepts?
    - How do we appropriately organize and model manipulatives for optimal learning?
2. Do teachers dismiss the importance of learning tools before students are ready to think abstractly?

- Why do many teachers dismiss the importance of learning tools before students are ready to think abstractly?
- How do concrete materials compare to more traditional teaching practices?
- How can teachers create a classroom that supports concrete learners while fostering those who think abstractly?

3. How do we bring "traditional" thinking teachers onboard with manipulative use?
   - Why do some teachers eliminate manipulatives prematurely.
   - Why do some teachers think negatively toward the use of manipulatives?
   - How do abstract thinking teachers reach and educate concrete learners?

Limitations and Assumptions

This study is limiting the research of manipulative use to grades three, four, and five. The author has chosen to restrict the research to these grades because she is observing a decrease in manipulative use when students get to the upper elementary grades. She will be focusing on the general manipulative use within the classroom instead of concentrating on individual manipulatives. She would rather see learning tools used to promote and strengthen the pictorial and abstract stages rather than have children enter these stages without scaffolding.

This paper and Freer Weiss (2006) assume that students of all abilities and at all levels can benefit from manipulatives in the classroom. The author has the expectation that classrooms should be centered on the children and their individual interests and learning styles. Lessons should be meaningful and relevant to the learner. This paper assumes teachers of grades three, four, and five want to teach according to their students' needs and in the manner that maximizes student learning. This paper assumes manipulative use will be used to foster and support student growth, individual learning needs, and academic achievement.

Definitions of Terms

Abstract Stage– During the abstract stage, or as Bruner (1966) calls it, symbolic representation, mathematical symbols are used in language to express the mathematical concept. Students demonstrate their understanding of the mathematical idea based upon the translation of experience into the use of language.

Child/Self Discovery – or as Dienes calls it, "building up personality" (p. 12) refers to the construction of the person (Dienes, 1960). A student notices and makes connections while engaging in active doings.

Concrete Stage– During the concrete stage, or as Bruner (1966) calls it, enactive representation, learning experiences are presented to children involving action based information to illustrate a mathematical concept. Students explore the concept using concrete tools in purposeful activity.

Manipulatives – physical, concrete objects and materials such as cubes, blocks, or tiles that can be used to support hands-on learning, help solve mathematical problems, and model mathematical concepts (Bouck & Flanagan, 2009; Puchner, Taylor, Donnell, & Fick, 2008).

Representation – "refers both to process and to product – in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself" (NCTM, 2000, p. 67).

Pictorial– Bruner (1966) identifies this system of representation as the Iconic stage. During the pictorial representation, pictures are used to illustrate models of the previous stage to represent a mathematical concept. Students visualize the mathematical concept using information stored from previous experiences at a pictorial level.

Subitization - “From a Latin word meaning suddenly, subitizing is the direct perceptual apprehension of the numerosity of a group” (Clements, 1999, pg. 400).

Virtual Manipulatives – are computer software tools that allow students to use simulations of physical manipulatives. “Virtual manipulatives are often similar to concrete manipulatives and often have the same names but are presented in an interactive manner through an online format or software environment” (Bouck & Flanagan, 2009, p. 187).

Summary Statement

The teacher of elementary-aged students should foster children's imaginations and guide them to self-discovery. Students should have the opportunity to explore hands on activities while being guided by their teacher. This research is aimed at exploring how manipulatives can be used to inspire a child’s self-discovery. It is also focused on diminishing the negativity some adults and teachers have surrounding manipulatives in the upper grades of elementary school while encouraging parents and teachers to gain confidence in teaching manipulatives to concrete learners.

Chapter 2: Summary of Research

Many aspects need to be considered when teaching with manipulatives. “There are probably as many wrong ways to teach with manipulatives as there are to teach without them” (Smith, 2009, pg. 115). The following research discusses the importance of manipulatives, classroom organization, understanding manipulatives as a tool to aid conceptual learning, teaching skills with manipulatives, using manipulatives to support student’s interests and learning styles, and the negative feelings conventional thinking parents and teachers have toward the use of manipulatives.

The Power of Child Discovery

Students who use manipulatives gain confidence in finding solutions to problems without relying on the teacher. Bruner states, “[it] goes without saying that, left to himself, the child will go about discovering things for himself within limits” (1961, p. 22). Tools can be utilized to discover an answer after the teacher poses a question. Johann Heinrich Pestalozzi was a prominent educational philosopher during the nineteenth century. He stated, “the child’s impulse to develop its reasoning powers through use is checked if the means by which one is trying to teach it to think do not attract and stimulate the intellect, but rather burden and confuse it” (Pestalozzi, 1931, p. 223). “It implies the child should not be given ready-made answers but should arrive at solutions himself and that his own powers of perceiving, judging, and reasoning should be cultivated, his self-activity encouraged" (Silber, 1960, p. 140). This is an illustration of child self-discovery and student-guided direction from the teacher. Teachers should make available many opportunities for children to experience firsthand. Richard W.

Copeland also believes “[t]he Teacher should provide a learning situation that will provoke the desired learning by the child” (1970, pg. 57).

Alcuin of York was a renowned scholar, poet, and teacher between the years of 735-804 C.E. Alcuin explained the purpose of his instruction was to, "draw out each pupil's latent powers, just as a man strikes out of a flint the fire that has all the time been hidden in it" (Cole, 1961, p. 128). Green explored Pestalozzi’s *Swan Song* and found, "[t]he inner impulse is already there, education has merely to furnish the opportunity and to offer guidance" (1969, p. 72). The teacher should guide students and ask them questions that foster their own abilities and reasoning. After the child decides upon an answer, they can share their reasoning with the class and teacher while demonstrating their way of thinking. Many students will learn from themselves and from their peers during discussions. Bill Jackson writes “[b]y sharing and discussing their solution methods, students develop metacognitive processes” (2010, p. 3).

Theories of Learning

Jerome Bruner (1966) developed three stages of learning which have impacted mathematical education. Bruner’s three stages of development are entitled enactive, iconic, and symbolic. Each stage of representation is integrated. Sequential order is less significant as they transform into each other.

The first of Bruner’s stages is enactive representation. During the enactive stage, students are actively engaged in experiences that form concepts stored in memory. The child’s experiences and motor activity help relate new information to past actions. A child may remember the sound a rattle makes after shaking it. This information stored in memory would prompt a child to shake the rattle in order to hear the sound (Bruner, 1966).

Bruner's (1966) second stage of representation is the iconic stage. The iconic stage is represented in the form of mental images and visuals of past activities. The child stores visual images in memory which helps them make connections to mathematical concepts.

Bruner's (1966) final stage of representation is the symbolic stage. During the symbolic stage information is stored and represented as symbols such as language and/or mathematical symbols. The three stages proposed by Bruner suggest a natural progression. Ages are not associated with Bruner's stages of development and his theory suggests adults may represent concepts at any of the stages.

Zoltan Dienes (1960) presented a series of principals pertaining to the use of manipulatives and abstraction. The four principals suggested by Dienes are the Dynamic, Constructivity, Mathematical Variability, and Perceptual Variability Principles. He also suggests most learning should take place individually or in a small group setting to allow for children's individual differences.

The first principal is the dynamic principle (Dienes, 1960). This principle suggests children should be given opportunities to experience hands-on activities. Games should be played with concrete materials to eventually build mathematical concepts.

The second principal is the Constructivity Principle. This principle suggests construction of the games should precede analysis. Dienes (1960) believes analysis is commonly absent from a child's learning until the age of twelve.

The third principal is the Mathematical Variability Principal. This principle suggests ideas concerning variables should be absorbed by experiences including the largest possible number of variables. An example of the Mathematical Variability Principal would be teaching several characteristics of a rhombus while using multiple teaching strategies and experiences (Dienes, 1960).

The fourth and final principal is the Perceptual Variability Principle. This principle suggests problems include distinct differences with common variables in order to challenge students and connect comparable problems. These four principals allow mathematical concepts to be presented using different resources to build up, in parallel, a sequence of similar tasks (Dienes, 1960).

Jean Piaget (1973) was a significant theorist who impacted mathematical education with his four stages of cognitive development. He believed intellectual development is attained by consecutive stages and levels. The four stages of development established by Piaget (1973) are the sensorimotor intelligence, pre-operational, concrete operational, and formal operational stages. Piaget's stages are developed in a continuous order of progression. Although ages are indicated for each phase, they can vary from one stage to another.

The first of Piaget's four stages is the sensorimotor intelligence stage. This stage occurs before a child's speech, usually before eighteen months of age. Piaget believes there is intelligence before speech, but no thought. A child is able to distinguish objects from themselves. They begin to identify themselves as a means of creating a result and begin to act intentionally. For example, a child understands when they shake a rattle it will make noise. Another example of a child grasping the concept of an object and acting deliberately would be a child that reaches for a toy hidden beneath a blanket. The child understands the object to still be existent even though it is removed from sight (Piaget, 1963).

The second of Piaget's four stages is the pre-operational stage. This stage occurs around the age of a year and a half or two years. Children in the pre-operational stage begin to characterize symbolical function, meaning they represent something with something else. The child begins to use language, play, gesture, and is able to classify objects within a group (Piaget, 1973).

The third of Piaget's four stages is the concrete operational stage. This stage occurs around age seven. This stage coincides with the beginning of elementary school for most children. Children begin to think logically and organize thoughts during the concrete operational stage. Children at this phase of learning are not ready for abstract thinking and can only think rationally about physical objects. Students become capable of understanding reversibility, the realization that actions can be reversed (Piaget, 1973).

The final stage of Piaget's stages of development is the formal operational stage. This stage begins around the average age of twelve to fifteen. During this stage of development the individual is able to think abstractly. They are able to scientifically test hypothesis and think hypothetically. In order to arrive at the formal operations stage of development, a person must first go through previous stages. Piaget (1973) believes each stage is significant for the success of the following representation.

Stages of Representation

Learning theorists, such as Piaget, Bruner, and Dienes, all seem to have similar views on educational psychology. These prominent theorists have studied children and have beliefs and systems which they feel represent the process of learning. The systems and stages of learning developed by each theorist show stages of representations from the concrete to abstract levels of cognitive development.

"Students who represent the problem in some way are more likely to see important relationships than those who consider the problem without a representation" (NCTM, 2000, p. 206). The Concrete-Pictorial-Abstract instructional approach focuses on a learning structure that leads a child through cognitive learning. Uribe-Flórez and Wilkins state that "Mathematical manipulatives offer students a way of understanding abstract mathematical concepts by enabling them to connect the concepts to more informal concrete ideas" (2010, p. 363).

The concrete stage focuses on a child's experiences and active engagement with physical manipulatives. This stage allows students to scaffold and create new connections with concrete objects relating to mathematical concepts. "The use of concrete material in the classroom has for one of its purposes the building up of mathematical imagery" (Dienes, 1963, p. 115).

Pictorial representation is a child's understanding of the concrete stage through images. Material is stored in the form of pictures in a child's brain linking active memory to mathematical concepts. Bruner states "[i]mages develop an autonomous status, they become a great summarizers of action" (1966, p. 13).

The abstract stage is the last of the three representations discussed in this paper. Bruner describes "the third or symbolic system of representation, based upon the translation of experience into language" (1966, p. 14). Students are able to understand mathematical concepts through symbols and are able to explain their understanding linguistically.

Manipulatives can help a child at any developmental level to move cognitively through concrete, pictorial, and abstract representations. "The materials provide experiences that help build clearer mental images, thereby leading to great understanding of abstract ideas" (Freer Weiss, 2006, p. 239).

Using Manipulatives to Support Learning

Children need hands on learning experiences to ensure learning (Copeland, 1970). Freer Weiss explains "[w]hen students can discover concepts through guided lessons and experience the process through multiple senses, they can structure and organize their knowledge for genuine learning" (2006, p. 242). Schwartz states "[o]ne of the most important ways to discover young children's emerging mathematical skills and understanding is by watching them in action and listening to their conversation" (2005, p. 5). Learning tools within the classroom help students connect physically and link the experience to abstract mathematical concepts. Freer Weiss states "[m]anipulatives can be used to make concrete representations of concepts that can be experienced by the senses" (2006, p. 239).

It is important for the teacher to understand the use of manipulatives and to teach them in a way that connects the children's cognitive thinking to mathematical concepts. Supplying children with educational tools does not automatically lead to student understanding, they need to be modeled and connected to student learning (Puchner et al., 2008). Smith states "[t]he key is whether the child understands the process, rather than simply moving cubes and rods around on a mat" (2009, p. 115).

“[Teachers] need to know and understand the content that they will teach to children, but they also need to hold a deeper understanding if they are to attend to their students’ ways of reasoning” (Philipp, 2008, p. 19). Professional development is crucial for teachers to continue to support their mathematical learners in their classroom. National Council of Teachers of Mathematics (NCTM) (2000) states “[teachers] must continue to learn new or additional mathematics content, study how students learn mathematics, analyze issues in teaching mathematics, and use new materials and technology” (p. 370). “[U]nless teachers are able to take part in ongoing, sustained professional development, they will be handicapped in providing high quality mathematics education” (NCTM, 2000, p. 370).

Organization

It is important to be organized when teaching with manipulatives. Boggan, Harper, and Whitmire (n.d.) state “[m]anipulatives can be extremely helpful [to] young children, but they must be used correctly. Children must understand the mathematical concept being taught rather than simply moving the manipulatives around” (p. 3). An educator needs to be prepared and lessons should be well thought out in advance. "The child needs an organized and structured world rather than an unorganized or unstructured one" (Maslow, 1954, p. 40). Students need daily routines, organization, and structure in order for them to carry out their daily responsibilities without worry. Meryem Yilmaz-Soylu and Buket Akkoyunlu state “every learning environment may attempt to raise successful students, but will not achieve the desired results if several essential elements are not considered in the instructional design process” (2009, p. 43).

Manipulatives need to be used frequently within the classroom for students to respect them as educational learning tools. Freer Weiss states, “students must be familiar enough with the manipulative materials that the use of them does not create an additional layer of frustration in the learning process” (2006, p. 241). Moyer and Jones state “when students see the materials used daily in the mathematics lessons, they will appreciate the usefulness of these materials for constructing meaning” (1998, p. 36). Some teachers do not take the time to effectively model manipulatives leaving the students confused about the educational purpose behind them. “When students are taught to use manipulatives as tools, they will be less likely to use them as toys” (Moyer & Jones, 1998, p. 36). Teachers will often send mixed messages by conveying that manipulatives are unimportant by using them only as novelties and for rainy days (Moyer & Jones, 1998). “If the student does not easily identify the purpose of the manipulative, it is no longer a tool but a distraction” (Freer Weiss, 2006, p. 241).

Rote Learning/Memorization

Many teachers and adults today can remember rote learning and memorization from their educational experiences. Students need concrete representations to help them grasp the concept being taught. Rote learning and memorization concentrates on procedures and techniques that ultimately lead the student to the correct response without any actual understanding of the concept. Memorization becomes a meaningless motion and disrupts the foundation of learning (Moyer & Jones, 1998). While a child relies on techniques, rules, and procedures to comprehend mathematics, the subject matter becomes more problematic (Moyer & Jones, 1998). “Students who have experienced rote learning may have serious deficiencies in such math strands as classifying, ordering, graphing, patterning, and problem solving" (Smith, 2009, p. 24). Students who rely on rote learning may also have difficulty depending on themselves and working independently. "By teaching ready-made rules and procedures, we teach children conformity, obedience, and dependence on adults for the correct answers" (Moyer & Jones, 1998, p. 4).

Using Manipulatives to Motivate and Engage students

Activities and lessons throughout education should be student centered, and they should also delve into their individual interests. A teacher's position is to make learning meaningful and relevant to one's life. "The kinds of experiences teachers provide clearly play a major role in determining the extent and quality of students' learning" (NCTM, 2000, p. 21). Giving children the opportunity to be involved while highlighting their interests will give them ownership of their education.

Education should be an adventure that allows students to explore learning by uncovering secrets to learning and their understanding. Waite-Stupiansky and Stupianksy says "[t]he challenge is to create situations whereby the manipulatives are used for uncovering, not just discovering" (1998, p. 1). Teachers should direct students through educational experiences and understanding. Children should be given the chance to experience multiple solution paths. Many research studies have discovered the use of manipulatives to be beneficial to classrooms and students who use concrete learning tools often outperform those who do not (Cain-Caston, 1996). Hands on activities give students opportunities to stay focused and interested in the given task nurturing learning and increased understanding. "If the mathematics studied in grades 3-5 is interesting and understandable, the increasingly sophisticated mathematical ideas at this level can maintain students' engagement and enthusiasm" (NCTM, 2000, p. 143). Giving a child manipulatives will engage the student in the topic being discussed. Using manipulatives can connect learning to real-life situations making education more relevant to the student. "Such real learning, based on first-hand experiences in the physical world, places mathematics in the realm of something that is fun, that can be enjoyed and that can be understood" (Copeland, 1970, p. 266).

Virtual Manipulatives

Technology continues to grow and is becoming a significant part of today's society. "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p. 11). With the popularity of technology and NCTM recommendations concerning concrete learning tools, teachers have become aware of virtual manipulatives and their usefulness in the classroom. "Virtual manipulatives are often similar to concrete manipulatives and often have the same names but are presented in an interactive manner through an online format or software environment" (Bouck & Flanagan, 2009, p. 187).

Bouck and Flanagan (2009) found that students are able to use virtual manipulatives on a computer almost the same way they would use hands on learning tools. Students and teachers alike are attracted to the appeal of virtual manipulatives. If supplies are limited or funds are lacking, a computer and internet connection could ultimately hold all manipulitives needed to address concrete learners at different stages of learning (Bouck & Flanagan, 2009). Bouck and Flanagan (2009) along with Burns and Hamm (2011) discovered students are attracted by the engaging, interactive, and immediate feedback technology allows. Their research also shows teachers can monitor students' progress and permit students to become more independent with the use of instant feedback while allowed to work at their own pace without feeling pressure from classmates (Bouck & Flanagan, 2009; Burns & Hamm, 2011). Teachers enjoy the use of virtual manipulatives and the lack of clean up that usually comes with physical learning tools (Burns & Hamm, 2011).

The use of virtual manipulatives is a great way for student's to interact with learning tools and give them experiences to connect to mathematical concepts. Research shows that the use of virtual manipulatives and/or physical learning tools has the potential to benefit students and teachers while strengthening mathematical concepts (Bouck & Flanagan, 2009; Burns & Hamm, 2011). However, as Bouck & Flanagan state "[b]oth virtual and concrete manipulatives are only beneficial tools if teachers know how to integrate them into their teaching" (2009, p. 190).

Eliminating Manipulatives Prematurely

"The frequency with which teachers use manipulatives in their classrooms differs between the elementary, middle, and high school levels" (Uribe-Flórez & Wilkins, 2010). Uribe-Flórez and Wilkins (2010) discovered research associating the decrease of concrete learning tools as the grade level increased. Some teachers feel manipulatives are only for young children and the older the child the less likely they need to use learning tools to solve abstract ideas. Zoltan Dienes believes "[a]s the child grows in mathematical maturity, he will have less and less cause to develop new mathematical concepts at first hand, i.e. directly from experience" (1963, p. 124). However, it is important to continue to supply children with experiences in order for them to grow conceptually.

Not all children spend the same amount of time on a particular stage and a teacher needs to remember not all children will be ready for the next stage of learning at the same time. “Using manipulative materials is one way we can help children build these structures at all stages of development” (Freer Weiss, 2006, p. 239). It is important for a child to comprehend mathematical concepts at a concrete and pictorial stage before moving on to the abstract stage. Copeland states “[i]f the child is unable to solve such a problem for a grouping of physical objects, then you can expect the same problem to occur as he attempts to solve a similar problem in the abstract world of number” (1970, pg. 98). Jackson declares “by spending time on concrete manipulation and pictorial representation, students are able to internalize and visualize mathematical concepts” (2010, p. 3). A teacher can aid a student's academic and social growth by investigating a child's learning style and understand student stages of learning.

Since children learn differently at different levels, it is crucial that physical objects are available for those not ready for the pictorial and abstract representational stages. Copeland states “[c]hildren must develop mathematical concepts from operations they perform on physical objects” (1970, p. 266). Dienes explains “pupils getting habitually ‘wrong answers’ are usually the ones whose understanding has not kept pace with the growth of the structure” (1960, p. 20). If we move a child beyond the concrete level of representation before they are ready they may never fully grasp the mathematical concept being taught. They may develop tricks and procedures to arrive at the correct answer without understanding why (Dienes, 1960). “Until we are certain that a child has made a fully valid abstraction, we might be foolish to rush in by suggesting symbolism.

This tends to freeze the abstraction process in the stage in which the symbols were introduced" (Dienes, 1963, p. 119).

Students having the opportunity to use manipulatives in the classroom will be able to gather experiences and apply them to future concepts. Students at the concrete representational level can use manipulatives and apply their experiential knowledge to the pictorial and abstract stages. Dienes states "[t]he use of the concrete material in the classroom has for one of its purposes the building up of mathematical imagery. Such imagery, once built up, can be manipulated without the aid of concrete objects" (1963, p. 119). Students will gradually move through the representational stages as they feel comfortable. Teachers should allow the child to move throughout the stages without pressure to eliminate stress, confusion, and frustration. Dienes also advises, "[c]hildren's own symbols will be the surest guide to the teacher in letting him know whether the bridge from the concrete to the abstract has been fully crossed" (1963, p. 118).

Eliminating Negative Stereotypes Linked to Manipulatives

Many teachers harbor negative feelings toward the use of manipulatives in the classroom. Teachers often complain that manipulatives are cause for student distraction, trouble, and frustration (Freer Weiss, 2006; Tooke, Hyatt, Leigh, Snyder, & Borda, 1992). Some teachers believe using manipulatives in a lesson takes up too much time and presumes students perceive instruction as "play time" (Tooke et al., 1992). Teachers' views and beliefs concerning manipulative use were found to be linked to their own ideas and experiences in mathematic's instruction (Moyer & Jones, 2004; Uribe-Flórez & Wilkins, 2010).

"In the past teachers were trained to teach mathematics through the use of procedures, rules, and algorithms" (Freer Weiss, 2006, p. 239). They were taught traditional algorithms and arithmetic processes rather than deriving understanding of mathematical concepts. According to Philipp, teachers "have little motivation for considering that the way they learned mathematics may not be the way they need to learn to teach mathematics" (2008, p. 12). Teachers who were taught symbols and procedures through the use of memorization and rote learning have difficulty explaining the meaning behind mathematical concepts. Freer Weiss states "[t]here is a disconnect between the training that many teachers have received and what current research suggests are better teaching methods" (2006, p. 239). Schorr, Firestone, and Monfils also state, "without a deeper understanding of the mathematics … they will continue to approach mathematics instruction as the transmission of an external body of knowledge rather than the creation an inquiry-oriented environment in which students explore and build mathematical ideas" (2001, p. 10). Tooke, Hyatt, Leigh, Snyder, and Borda (1992) explains if teachers "can be shown manipulatives for teaching their students, but shown in such a way that the focus is on the mathematics involved, perhaps more of them will be receptive to the materials and their use in the classroom" (p. 62).

Not all teachers who were taught traditional algorithms and procedures dismiss the idea of teaching with physical learning tools. The National Council of Teachers of Mathematics (NCTM, 2000) states there are many publications highlighting manipulative use, however, the specifics of how to effectively use them are still not understood clearly (Moyer & Jones, 2004). Many teachers are lured in by the appeal of manipulatives, but do not use them in a way that encourages student understanding (Schorr, Firestone &

Monfils, 2001). Teachers may be unfamiliar with manipulatives and the way to connect understanding to mathematical concepts.

Manipulative Studies

Many studies have been conducted to show the impact of physical learning tools among children within the classroom. Research and studies based on concrete learning tools, virtual manipulatives, levels of cognitive development, etc. are beneficial as educators explore students' learning styles. These studies help teachers understand children and the best ways of teaching to their ability in order to achieve academic success. Teachers can then modify their teaching to help children learn mathematical concepts based on what the research suggests.

Bruner and Kenney (1965) observed Dienes and his Harvard assistant while teaching a small group of children, four eight year olds for six weeks. Dienes and his assistant promoted independent problem-solving. The purpose of this study was to observe the eight children while identifying the stages in which they grasped mathematical ideas. Guiding questions were presented to the students in order to bridge their thinking and connect meaning to the concept before readily discovering independently (Bruner & Kenney, 1965). After completing the study, Bruner and Kenney suggest that learning mathematics "begins with instrumental activity, a kind of definition of things by doing. Such operations become represented and summarized in the form of particular images" (1965, p. 56). They go on to explain "with the help of a symbolic notation that remains invariant across transformations in imagery, the learner comes to grasp the formal or abstract properties of the things he is dealing with" (Bruner & Kenney, 1965, p. 56).

A study conducted by Burns and Hamm explored the relationship between virtual and concrete manipulatives. Ninety-one third and fourth grade students participated in the study and were assigned to work with either virtual manipulatives or physical learning tools while working with fractions. They were given a pre-test and a post-test to measure their growth.

Results show "both concrete and virtual third-grade manipulative groups showed improvement from pretest to posttest, [however], there was no statistically significant difference in scores of third graders' fraction knowledge" (Burns & Hamm, 2011, p. 258-259). Burns and Hamm (2011) posited these results were the effect of students having higher pretest scores and that it was not an introductory lesson, but a review of fractions. Students in both third and fourth grade using either virtual or concrete manipulitives showed enthusiasm and were excited to use the tools in conjunction with the fraction lesson. "Results suggest the use of either virtual or concrete manipulatives or a combination of both to reinforce math concepts" (Burns & Hamm, 2011, p. 259).

Uttal, Scudder, and DeLoache (1997) conducted a study to "clarify the relation between children's comprehension of symbolic relations and their understanding of manipulatives" (p. 39). They were asked to locate a toy hidden in a room using a simple scale model of the room. Children were shown the scale model; the toy that was hidden in the larger room was depicted in the room's model. Children then entered the larger room and were asked to locate the toy using information from the model. "To succeed, children have to detect and exploit the 'stands for' relation between the two spaces" (Uttal, Scudder, & DeLouche, 1997, p. 41). The study involved children 30-38 months of age.

Many children had difficulty retrieving the toy from the larger room when relying on the smaller scale model. "They almost always failed the symbol-based retrievals averaging around 20% correct searches" (Uttal et al., 1997, p. 41). Children averaging six months older were much more successful at finding the toy. "The younger children failed to appreciate the relation between the model and the room and that this relation was relevant to finding the toy" (Uttal et al., 1997, p. 41-42).

After the study was conducted Uttal, et al. (1997) "suggests that to us any object, including a mathematics manipulative, as a symbolic representation, children must appreciate the relation between the object and its referent" (p. 45). The researchers recommend many implications be considered before working with manipulatives within the classroom. First, "[m]anipulatives are not a panacea," second, "[t]eachers must consider children's interpretation of manipulatives," third, "[m]anipulatives cannot be a substitute for instruction," and lastly, "[m]anipulatives must be chosen and used carefully" (Uttal et al., 1997, p. 50-51).

Chapter 3: Interpretation

Many teachers use and find manipulatives beneficial in the classroom. Chapter two examines many findings related to learning tools and the role they serve in the classroom to help aid concrete learners become confident abstract thinkers. This paper will continue to review the research on manipulatives and further discuss why they can be advantageous to mathematics students in grades three, four, and five.

Child Discovery vs. Adult Guidance

Concrete materials within the classroom serve as a hands-on guide to learning. Students may need to discover the concept of learning through the manipulation of physical objects building a foundation for subsequent stages of learning. Bruner (1966) believes children have a role in the learning process and often times discover learning independently while being observed and guided by an adult. Schwartz (2005) agrees in order to determine children's evolving mathematical understanding and skills, one of the most important ways is by watching and observing students in action while listening to their thoughts. Piaget (1947) on the other hand, trusts adults and educated peers make a difference in a child's educational growth. Freer Weiss (2006) agrees that concrete tools alone do not influence academic progress, but is rather the result of effective teaching. Freer Weiss and Piaget believe in adult guidance while Bruner and Schwartz trust in student discovery. The author of this paper interprets these opposing views to vary based on class size, stages of student development, and the task assigned. She agrees with Freer Weiss (2006) and Piaget (1947) who believe manipulatives need to be supported by an effective teacher who can carefully guide student learning. A teacher must determine the surrounding factors within a classroom and perceive student indications of whether independent, guided, or a combination of the two is needed for academic success.

During Bruner and Kenny's 1965 study, children were observed and guided through concrete exploration. Although Bruner believes student discovery is key, this study demonstrates guided instruction. The author of this paper concludes that student discovery is significant, as is flexible.

Supporting Learning with Manipulatives

Manipulatives can serve a positive role in the classroom if they are used correctly as teaching tools. Teachers need to be familiar with manipulatives and understand how to teach with them effectively while connecting the experience to mathematical concepts. It is important for the educator to teach using a variety of educational tools to reach each individual student. Not all children learn the same way or at the same pace, and it is crucial for the teacher to present a lesson addressing multiple learning styles with the use of tools that represent mathematical concepts.

Cain-Caston (1996) found that manipulatives used in the classroom are beneficial and students who use them out perform those who do not. Burns and Hamm (2011) discovered no statistical significance in third and fourth grade student fraction knowledge after being taught with either physical or virtual manipulatives. They did, however, see an improvement from pre and post tests and support the use of manipulatives in the classroom. The author of this paper understands from these research studies that physical and virtual manipulatives both show positive impact on student learning and are helpful additions to the classroom when used correctly.

Both NCTM (2000) and Philipp (2008) advocate teacher training and professional development to continue the development of mathematical education. They both believe it is crucial for teachers to receive the training needed to understand how to teach with manipulatives and connect students to mathematical ideas. Freer Weiss (2006) believes there to be a significant gap between teacher training and better teaching approaches. She also explains that teachers working with manipulatives, who do not have the skills needed to appropriately teach with concrete tools, will fail to be beneficial. This paper agrees with the information supporting professional development, manipulatives alone cannot successfully connect students to mathematical ideas without the competence of the teacher.

Organization

Teachers implementing concrete learning tools need to take time to model and efficiently establish the significance of manipulatives as learning tools. Research shows that many teachers feel physical objects are more of a hassle than helpful in the classroom. Freer Weiss (2006) believes if children are not acquainted with daily use of manipulatives they become a distraction rather than an aid to education. Tooke et al. (1992) found that teachers have a tendency to use manipulatives during down time and therefore students have the misconception that concrete tools are toys. When children lose the meaning behind the manipulatives, they can often treat them poorly leading to mischievous behavior and trouble. Dienes (1960) agrees that manipulatives need careful organization and modeling. He explains that children need an assortment of experiences that lead children to establish mathematical concepts. He warns that numerous experiences are needed to establish such concepts in order to ensure generalization, not just association, occurs. The author interprets from this information that concrete learning tools need to be established throughout daily routines and used frequently. The author also understands these tools need to be modeled and demonstrated to connect them to mathematical concepts. This author agrees with Freer Weiss (2006) and Dienes (1960) that concrete tools are just objects needing an efficient teacher to connect learning experiences to mathematical concepts.

Stages of Representation vs. Rote Learning/Memorization

The representation stages of cognitive development have been researched by many theorists. Jean Piaget, Jerome Bruner, and Zoltan Dienes are a few of the well known theorists who have impacted mathematical education with their theories of learning. They all have their individual philosophies regarding the stages of learning and development. Although these theorists have identified the stages differently, this study combines them to represent the stages of learning with the terms concrete, pictorial, and abstract as this author feels they relate to the main focus of their ideas of development.

The concrete stage offers children hands-on activities with physical learning tools. Bruner (1966) calls this stage the enactive representational stage. Copeland (1970) has found children to have the desire to experience physical objects because of the fact they are a part of the physical world. He also believes hands on activities with concrete objects are essential for learning. The concrete stage is the foundation for the remaining stages and information stored from experience support the remaining stages.

The pictorial stage builds upon the concrete stage. Bruner (1966) calls this stage the Iconic Stage. During this stage children have visual images of what they did during the concrete stage locked into their memory. They are able to draw pictures to represent mathematical concepts. Bruner (1966) believes the pictorial stage is the visual depiction of a child's actions.

The abstract stage is the final representation of learning described here. A child that understands mathematical concepts is able to understand symbols and communicate their understanding through language (Bruner, 1966). Bruner (1966) identifies this stage as the symbolic stage while Piaget calls it the formal operational stage. Smith (2009) agrees with Piaget and his stages of cognitive development. She believes the system of representation to focus on the child's process of thinking rather than just the answer, quality experiences, delaying adult-like thinking before children are ready, and portraying the teacher as a guide to learning.

Rote learning and memorization are techniques that help children gain information quickly. The practice of teaching students tricks, procedures, and rules rather than meaningful experiences often lead children to become dependent on ineffective rote learning (Moyer & Jones, 1998; Smith, 2009). Dienes (1960) warns students having difficulty comprehending mathematical concepts often resort to rote learning and memorizing facts to keep pace with others.

This study interprets the information supporting stages of representation to be most beneficial to educators because it helps them understand the system of learning and how to help a child at each representational level. This author understands this to be the best way to assist children in learning and recognize rote learning and memorization to be poor tactics to assist students' educational growth. She gathers the information to suggest that rote learning and memorization do not support student learning and does not help them connect their ideas to mathematical concepts.

Manipulatives as a Motivator rather than a Distraction

The use of manipulatives within the classroom can have a positive effect on student behavior and often times can motivate students. Moyer and Jones (1998) believe student attitudes and intrinsic motivations have the possibility of improvement with the use of manipulatives. With students becoming more familiar with technology and media, virtual manipulatives can be a motivation for students that thrive on computer software and interactive media on the web.

Manipulatives need to be modeled appropriately and used as a tool to connect experiences to mathematical concepts. When physical learning tools and virtual manipulatives are used within the classroom it should be the hands-on experience that motivates students, not because they are seen as toys. Moyer and Jones (1998) found that teachers would take manipulatives away from students as punishment if student behavior was not acceptable. This information tells this author that manipulatives need to be used frequently and not depicted to children as a reward that can be taken away for bad behavior. She finds this evidence interesting because manipulaives frequently used ultimately serve as an intrinsic motivator and should not be used as a reward system.

Eliminating Manipulatives from the Classroom Prematurely

Copeland (1970) fears educators are often teaching mathematical concepts before the child is ready to comprehend them. He finds children must expand upon mathematical ideas from actions they experience with physical objects. Dienes (1963) agrees and warns teachers not to be foolish and rush children to abstract thinking before they have made a valid connection. He also explains that hastening the process will only stop a child from continuing to the next stage. Many students stay concrete learners well into high school and college (Epstein, 1998).

This study interprets these findings to be important because students need the support from their teacher to guide them through the instructional stage they are in rather than rushing the process with symbols before they are ready. This author finds this information significant because students need to be taught at the stage of representation they currently are in regardless of their age or surrounding pressures. Teachers need to understand the levels of representation and guide their students through cognitive understanding without weakening their understanding by quickening the process.

Chapter 4: Findings and Recommendations

Having read many literature studies and research articles with professional theories focusing on the use of manipulatives, this study will continue to explore manipulatives within the author's own teaching experiences. She will further discuss manipulatives as they relate to her classroom experience with learning tools. From the literature, studies, and experiences gathered on the use of manipulatives she will determine notable needs and recommendations for further research.

Author's Experiences

The district in which this author works is located in northern Minnesota on a Native American reservation. She works at an elementary school which has approximately 600 students enrolled in Kindergarten through fifth grade. Of the 99% Native American and 1% African American enrollment, 89% receive free and reduced lunch. The elementary has 14% of its student population receiving services for special education and students considered English Language Learners (ELL) are among 6% of students.

The author's title is a Response to Intervention Specialist (RTI). She is one of ten RTI specialists who work with students in the elementary. She work with small groups of children in grades first through fifth who are not performing at grade level, but are not eligible for special education services. The RTI program services nearly half the student population in this school.

The author takes two to four students from the classroom for half hour periods to work on material which will support them in the regular classroom. Many children are pulled from the classroom to work in a smaller group by many RTI specialists leaving the regular classroom teachers with a small group on which to focus. Students are tutored within a smaller group in an environment that allows them to make mistakes, ask questions, work at their own pace, and have the attention and guidance of the teacher. Group size is established based upon the empirical need shown by student data; smaller groups afford more support along with more individualized intervention. The author teaches students in both reading and mathematics throughout the day. She tutors four reading groups and five groups in mathematics. Her five mathematics groups include every grade at the elementary level. Students are placed in tutoring with peers performing at a comparable level based on data from Measure of Academic Progress (MAP) scores, Assessment and Data Management for RTI (AIMSweb) benchmark and progress monitoring, and Minnesota Comprehensive Assessment (MCA) scores. Students are then progress monitored throughout the year in order to make grouping arrangements and decisions regarding their educational support. Students may be exited from the RTI program if they are performing at or above grade level. They may also be referred to the Student Support Team (SST) for further action and intervention if their progress does not show improvement.

The purpose of the RTI program is to offer students intervention services along with remedial support. A student may be missing crucial mathematical concepts preventing them from understanding grade level material that could require them to think beyond their current cognitive level. The goal of the RTI program is to prevent academic failure with early intervention, frequent progress monitoring, and the use of research-based instructional interventions.

Manipulatives in The Author's Classroom

Students in the author's classroom are often in the concrete and pictorial levels of cognitive thinking, and very rarely are seen in the abstract stage. Her goal is to offer them as many experiences to support them in their mathematical thinking. In order to nurture their mathematical growth, she has a number of concrete learning tools available for their use. Some of the manipulatives she has available are Diene's blocks, beaded counting strings, clocks, currency coins and bills, geometric shapes, fraction circles and pizzas, fraction chips, Unifix cubes, etc.

The author uses Diene's blocks and Unifix cubes frequently. Diene's blocks are cubes arranged in ones, tens, hundreds, and thousands. Students often call the ones units, tens longs, hundreds flats, and thousands blocks. Diene's Blocks help my students represent the meaning of place value and number sense. Students are able to use them during addition, subtraction, multiplication, and division type problems. Unifix cubes are colored cubes that can be linked together helping students with addition and subtraction type problems. She also created counting strings to have available in her classroom. The counting strings have colored beads changing colors on every tenth bead. Each string has one hundred beads and students have clothes pins they can use to mark separations. They use the counting strings when counting by tens, adding, and subtracting.

Clocks, coins, and bills are representations of real life objects that help students become familiar with notions relating to time and money. The author's students love working with coins and bills because it gives them the opportunity to participate in life experiences.

Geometric shapes are available for my students to help them differentiate between two and three-dimensional shapes. They are also available to assist them in situations where perimeter, area, volume, and surface area are involved. The author's students also have the opportunity to make their own three-dimensional shapes with marshmallows and toothpicks. This gives them a hands on activity which helps them identify edges, vertices, and faces within an object.

Fraction manipulatives are essential for students beginning to comprehend rational numbers. Fraction circles and pizzas help students understand part to whole ratios, equivalent fractions, and fraction operations. Fraction circles/pizzas allow students to work with something with which they are familiar and this helps them when entering the pictorial stage. Chips aid students with fractions of a set and are helpful when partitioning a group of objects.

Classroom Organization

The author usually introduces the students to the physical objects while connecting them to mathematical concepts in a lesson. Once given the instructional message with the use of learning tools, the children are free to use them with any mathematical situation they feel needs modeling. Not all the students within the group want to use concrete tools, but will instead choose to depict the tools in drawings to assist them in finding solutions. These students are showing they are ready for the pictorial stage of learning. Multiple solution paths are encouraged in her classroom and supported with a variety of different mathematical tools for learning. Groups using a variety of different solution paths will learn from each other and connect mathematical ideas to explanations presented by others.

Concrete tools are always available and used frequently so students are not confused of their purpose. The author discourages students from turning manipualtives into toys. She does not like to see students distracted by manipulatives. If the author sees students distracted from the task of the lesson because of concrete tools, she tries to represent or introduce another manipulative in its place. The author models the lesson based around the manipulatives that best support the lesson and demonstrate how to use them effectively.

Student Motivation

The author has found through informal observation that manipulatives offer students hands on activities that better engage them. Students in her classroom take responsibility and show ownership of their solutions more so with the use of manipulatives. Students become motivated to use the manipuatives in order to discover individual results. Students become inspired to create their own solution paths using manipulatives in order to share their thinking with the group.

NCTM (2000) recommends the use of virtual manipulatives to enrich teaching and improve student learning. The author is fortunate enough to take her students into the computer lab once a week to work with virtual manipulatives. The programs used most often are Education City, NCTM's Illuminations, and National Library of Virtual Manipulatives. These educational programs offer her students an opportunity to work with technology while practicing with learning tools they are familiar with from the classroom setting. Virtual manipulatives excite her students while giving them a chance to experience a change of pace from daily instruction in the regular classroom. Students are enthusiastic about the prospect of working with computers and like the interaction of the programs and immediate feedback.

Classroom Benefits of using Manipulatives

This author has seen many benefits of using manipulatives in her classroom. She finds her students are more engaged, focused and motivated when working with concrete learning tools. A student-interest survey often reveals students coming into the classroom dislike mathematics. She finds herself relating to these students and the feeling is all too familiar to her mathematical experiences during school. Manipulatives offer them a different perspective on the often times laborious subject of mathematics. The author observes students in her classroom frequently forgetting the difficulty of the activity when using concrete tools.

This study finds manipulatives offer students in the concrete level of representation the hands on activities needed to offer experiences required to support this stage of learning. Manipulatives help students experience firsthand activities that help them understand mathematical concepts they would otherwise have struggled with. It gives them an opportunity to visualize mathematical ideas to guide them to solutions instead of using techniques, tricks, and procedures. The author sees students at the concrete level easily transition to the pictorial stage of learning with the help of manipulatives. They adapt more easily when having had the concrete experiences to visualize and illustrate.

The author's students enjoy working with manipulatives or depicting them in illustrations rather than solving problems without support. Students in her classroom do not feel comfortable thinking abstractly about many situations and find solace in working with tools to help them find solutions. She observes her students exhibiting less anxiety and pressure with the use of manipulatives.

The Need for More Research

A conclusion of this study is that manipulative research has strongly focused on younger children and this focus demonstrates a significant need for more research concerning older elementary children through adulthood. Much of the research focusing on older students relying on manipulatives were children considered special need students or students within remedial programs. This author would like to see more research done on students at the upper elementary, middle school, and high school grade levels, within the regular classroom, to determine the benefits of manipulatives and their effect on students.

This study also concludes there needs to be more advocating and support for teacher's learning how to teach using manipulatives and successfully connect them to mathematical learning. Professional development and continued educational growth is very important for teachers using manipulatives. The research discussed clearly communicates a need for teacher trainings and professional development programs pertaining to learning tools. During the author's research she discovered many teachers having apprehensions towards the use of manipulatives (Tooke et al., 1992). Such programs and initiatives will likely reduce the anxiety many teachers feel toward teaching with concrete learning tools.

Recommendations and Future Use

This paper recommends the use of manipulatives at every grade level especially grades three, four, and five. This author discovered through research and experiences that students learn at different levels and rates. Students in grades three, four, and five may not be ready for the pictorial and abstract stages and may be deprived from the tools necessary to help them be successful. Students that are prematurely forced from the concrete stage into the pictorial and abstract stage may be missing fundamental understandings needed to fully grasp the mathematical concept. Without concrete experiences students may be forced to learn procedures, tricks, and actions that will lead them to acceptable answers without any mathematical understanding.

This author plans on continuing the use of manipulatives within the classroom and Response to Intervention program to assist students through the levels of cognitive development. She plans on having concrete materials available for students at all grade levels including those in grades three, four, and five. She hopes to participate in professional development programs relating to the use of manipualtives in order to strengthen her skills of teaching with concrete tools. The author of this paper anticipates sharing her research and findings with colleagues during common planning time, professional learning teams, and staff development days in hopes of encouraging them to look more closely at manipulatives and the many benefits they serve children in the classroom.

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