School of Graduate Studies
Bemidji State University
1500 Birchmont Dr NE, # 27
Bemidji, MN 56601-2699
218-755-2027

MANIPULATIVES IN MATHEMATICS INSTRUCTION

by

Jessica Strom

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STATEMENT BY THE AUTHOR

Signed: _____

APPROVAL BY RESEARCH PAPER ADVISOR

THIS RESEARCH PAPER HAS BEEN APPROVED ON THE DATE SHOWN BELOW:

_____ _____

Dr. Glen Richgels, Date
Committee Chair
Professor of Mathematics and Computer Science

_____ _____

Dean, School of Graduate Studies Date

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This paper is a review of research pertaining to the use of manipulatives in middle and secondary school mathematics instruction. It covers the research on the rationale for using manipulatives, the psychology behind learning with manipulatives, the common mistakes when teaching with manipulatives, and the suggested process to follow when using manipulatives in the classroom.

There is a large amount of research and information on manipulatives in the classroom. One of the most prevalent topics addresses that the concrete characteristics of manipulatives allow students to progress through a natural learning process on their way to an abstract understanding of mathematical properties. They also provide an avenue for communication, which is a valuable resource in mathematics education. Common mistakes include not providing enough time, assuming the meaning behind the manipulatives is transparent to students, not helping students make the connection between the object and the mathematical concept, and not providing enough communication. The suggested process includes time given to students to work at a concrete level, then progress to a representational level, and finally to an abstract level. The teacher's role is to provide appropriate activities that bring students through the process and to pursue communication that reveals students' thinking and provides learning opportunities.

Approved by:

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Committee Chair Date

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Committee Member

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Committee Member

_____

Graduate Faculty Representative

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Chapter 1: Introduction

The use of manipulatives in mathematics instruction has become a passionate topic in my career. I have always felt that students can learn the mathematical topics present in our secondary curriculum. The problem was that I could not figure out how to get there. When I started teaching I inherited a bag of manipulatives called Algebra Tiles. I took them out for about half an hour, got confused, figured my students would too, and put them back into the cupboard. They stayed there for the next five years.

I continued to teach with the philosophy that I should be able to reach each student somehow. I had heard in my undergraduate teaching methods classes that when students learn kinesthetically, they learn better and even remember it better. I experimented with different ideas I had brainstormed, some worked and some did not. Then I had the great experience of going to a workshop where the presenters led us through some exercises using Algeblocks and pegboards. At that moment, the teaching floodgates opened as I saw the implications of what I had just experienced. I started trying new things in my classroom using manipulatives and found great results. Ever since then, I have searched for more ideas, refined what I have been doing already, and found that, although they are a great tool, teaching with manipulatives needs to be understood by the teacher in order to have the greatest effect. I decided the perfect topic for my research paper was the use of manipulatives in the classroom.

There are various methods of mathematics instruction. Using manipulatives is a popular method, but one that needs to be understood and executed correctly (Bright, 1986; Ball, 1992; Cobb, Yackel, & Wood, 1992; Thompson, 1994; Clements & McMillen, 1996; Boulton-Lewis, 1998; Jones, 2000; Kilpatrick, Swafford, & Findel,

2001; Moyer, 2001; Suh, & Moyer, 2007; Puchner, Taylor, O'Donnell, & Fick, 2008). It is the teacher's responsibility to make sure the students make the connections between the concrete manipulatives and the abstract manipulations. In order to do this, we need to understand how and why manipulatives help students learn and then how to help them make the connections.

Significance of the Research Problem

The world is changing and the students in the United States are behind when it comes to math (Lemke & Patrick, 2006). As a result, our education system is pushing for improvement. We need to change the way we teach mathematics to bring U.S. students to a level of competition with the rest of the world and to make it possible for all students to learn mathematics. Students have fewer and fewer physical experiences that allow them to develop fundamental number concepts, instead these experiences are replaced with technology (Keller, 1993). We need to address the lack of physical experiences with manipulatives where needed, but also utilize the technology students are familiar with to enhance their understanding.

All students should have the opportunity to learn the fundamental mathematics we teach in elementary and high school. However, as various tests results and classroom observations are showing, our students are missing out. Manipulatives can be part of the answer, but the use of manipulatives is not an automatic solution. They need to be implemented correctly. Knowing this can mean the difference between a waste of time playing with toys and a dynamic teaching tool that makes mathematics accessible to all students.

Statement of the Problem

Manipulatives can be a great tool, but they need to be used appropriately. If not used appropriately, they can be confusing and, as some would say, just a toy. In order to really understand how to use manipulatives appropriately and effectively, it is important to understand how and why they work to help students understand mathematical concepts. This will also serve as justification for using manipulatives in instruction. There are many mathematics instructors who, not knowing the usefulness of manipulatives, shy away from using them or use them inappropriately.

Research Questions

How do manipulatives help students learn mathematical concepts?

1. Is there a part of the learning process that manipulatives help where other methods might not?
2. What is the educational psychology behind using manipulatives?

Is the effect of the use of manipulatives the same for all students?

1. Are manipulatives useful for learning disabled students?
2. How do gifted and high achieving students do with manipulatives?

What are mistakes to stay away from when using manipulatives?

What process or methods should be used when teaching with manipulatives?

1. What makes this process/method effective?
2. When is it appropriate to use manipulatives?
3. Is there a specific time frame for each step in this process?

Limitations and Assumptions

I am limiting my research to middle and high school mathematics instruction. I am also limiting it to the use of physical and virtual manipulatives in instruction and not for use in games.

I am assuming all students can learn mathematics. I am assuming that there is a teaching method for every student, not a single method, but each student can be reached by some method. I am also assuming that teaching with manipulatives is not an art, but a method that can be learned, understood, and executed correctly by all teachers.

Definition of Terms

Physical manipulatives – usually referred to as "manipulatives", objects that can be physically touched and moved by the student. The objects and the manipulation of the objects represent abstract mathematical concepts (Kennedy, 1986; Williams, 1986; Moyer, 2001).

Virtual manipulative – "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard, & Spikell, 2002).

Concrete stage – the stage of learning in which students work only with the manipulatives to understand a mathematical concept, symbols are absent during this stage (Schultz, 1986; Sowel, 1989; Keller, 1993).

Representational stage – also called the "pictorial stage", the stage in which students use pictures in the same manner as manipulatives, the manipulatives may or may not be present. It is also the stage in which students may watch the manipulation of manipulatives instead of doing it themselves (Schultz, 1986; Sowel, 1989; Keller, 1993).

Abstract – the stage in which there are no manipulatives or pictorial help but symbols instead. Students manipulate the symbols using the math concepts learned from the previous stages (Schultz, 1986; Sowel, 1989; Keller, 1993).

Summary Statement

Manipulatives are tools used in mathematics instruction that, when used effectively, can hopefully help all students learn the mathematical concepts taught in elementary, middle, and high school. This research is aimed at understanding how manipulatives help in the learning process and what the appropriate methods are to ensure students learn the concepts and are able to bring their understanding to an abstract level.

Chapter 2: Summary of Research Sampling

The use of manipulatives has many different facets to consider. The research below discusses the reasons we should be teaching using manipulatives, how manipulatives help students learn math concepts, the mistakes teachers make when using manipulatives in their classrooms, and the process that should be used when teaching with manipulatives.

Why Use Manipulatives in Mathematics Instruction?

Why take the time to teach with the physical manipulatives and not just at an abstract level? Much research indicates that manipulatives are a worthwhile method of instruction (Kennedy, 1986).

To begin with, a well chosen manipulative mirrors the concept being taught and gives the students objects on which to act. They become active learners of concepts that may otherwise just be symbols (Heddens, 1986; Boulton-Lewis, 1998; Kilpatrick, Swafford, & Findel, 2001). In order for this action to be a learning experience, students must reflect on what their action did and what this means for the mathematics concept being learned. In this way, the manipulatives become tools for thinking and allow students to correct their own errors (Thompson, 1994; Clements & McMillen, 1996; Boulton-Lewis, 1998; Kilpatrick, Swafford, & Findel, 2001). This becomes extremely useful in situations where students may be self-conscious and unwilling to bring attention to themselves by asking questions (Moyer, 2004). Also, the contact with the manipulatives gives students a visual to help with their memory and recall of the concept (Boulton-Lewis, 1998; Suh & Moyer, 2007).

According to Resnick (1983) all learning is based on prior learning and experiences. Learning involves connecting new concepts to prior knowledge. Manipulatives can serve as a tool for teachers to link the students' experiences with the objects (prior knowledge) to the abstract mathematical concepts (new knowledge) the objects are representing (Kennedy, 1986; Cobb, Yackel, &Wood, 1992; Cain-Caston, 1996; Moyer, 2001; Kilpatrick & Swafford, 2002). This can also utilize students' prior knowledge in one representation to learn a new but similar representation, such as the multiplication of integers to the multiplication of polynomials using an area model (Bright, 1986; Balka, 1993). Ultimately, working with the manipulatives allows students to make the connections between the concepts and the algorithms and procedures used with those concepts (Balka, 1993).

As computers and calculators become more and more advanced and able to perform algorithms for us, we need to look at why we teach them in our school systems. The reason being that students still need to understand the procedures the technology is performing (Beattie, 1986). Evidence shows that manipulatives help students understand mathematical concepts and builds a foundation for understanding algorithms; a consequence is that they will also help with the mastery of the algorithms (Kennedy, 1986; Moser, 1986; Beatie, 1986; Jones, 2000).

A large impact made by the use of manipulatives is the improvement of students thinking. They help students create an internal representation of the external concepts being taught (Puchner, Taylor, O'Donnell, Fick, 2008). They help students with their algebraic reasoning and relational thinking (Suh & Moyer, 2007). They also help advance

students to higher cognitive levels including analysis, synthesis and evaluation levels of Bloom's Taxonomy (Keller, 1993; Balka, 1993).

An important part of making manipulatives a success is the communication that takes place during the lesson. First, manipulatives make it possible for the teachers and students to communicate their thinking by giving them something at a concrete level (Ball, 1992; Thompson, 1994; Kilpatrick, Swafford, & Findel, 2001; Moyer, 2001). During this time, teachers can discover students' thinking and guide them to the concept at hand.

Virtual manipulatives are a version of the physical manipulatives, that are on the computer screen rather than a desk. The user moves the virtual manipulatives using the mouse or key strokes. One of the biggest arguments for using virtual manipulatives versus physical manipulatives is that they have the ability to connect the movement and action on the manipulatives to the symbolic notation simultaneously (Moyer, Bolyard, & Spikell, 2002; Suh & Moyer, 2007). This form of manipulative helps students make the connections between the physical and symbolic representations that they may not have otherwise because of the cognitive overload that can happen while students are trying to keep track of their actions on the physical manipulatives and the symbolic representations that go with those actions (Suh & Moyer, 2007). Because of this simultaneous picture and notation, the students are given immediate feedback and are able to self-check what their actions accomplish and enhance their theories in real-time, it also gives a guide to the algorithm being learned and allows them to see and use multiple representations for the concept being learned (Dorward, 2002; Suh & Moyer, 2007).

Other benefits of virtual manipulatives include their accessibility outside of the school as they are on the internet and often link to other helpful sites (Dorward, 2002; Moyer, Bolyard & Spikell, 2002). The user can also modify colors and configurations, which may enhance the understanding of the concept being learned (Dorward, 2002). Also, the use of virtual rather than physical manipulatives may be more accepted in the secondary setting. Reasons for this include high school students may view the physical manipulatives as "toys" instead of a learning tool, the technology may be more enticing to those students, and the virtual manipulatives may allow them to be more creative in their solutions (Dorward, 2002; Moyer, Bolyard, Spikell, 2002).

Studies by Dorward (2002) and Suh and Moyer (2007) indicate the use of virtual manipulatives or the combination of virtual and physical manipulatives improve achievement, cover as much content as teaching at an abstract level, and may even improve attitudes toward learning mathematics.

The same holds true for the exclusive use of physical manipulatives. Data shows students cover the same amount of content or more using physical manipulatives than those who are not using manipulatives (Raphael, 1989; Johnson, 1993). They also show that the use of manipulatives is related to gains in achievement, which is especially evident with low-achieving students (Threadgill-Sowder, 1980; Suydam, 1986; Sowel, 1989; Cain-Caston, 1996; Suh and Moyer, 2007). The result of this may mean the need for remediation later is no longer existent or at least at a very reduced rate (Moser, 1986).

How do manipulatives affect the special learners in the classroom, whether they are learning disabled, low-achieving or mathematically gifted? Learning disabled and low-achieving students have shown that a structured approach that allows the students to

be active in their learning helps them understand the concepts and algorithms and be successful; manipulatives give them the multi-sensory, active avenue of learning (Threadgill-Sowder, 1980; Thornton, 1986; Witzel, 2007). Gifted students also benefit from the use of manipulatives. Although they do not need to use them to the extent of others, the experience of using manipulatives allows gifted students to bring their thinking to higher levels (Moser, 1986; Thornton, 1986).

How Do Manipulatives Help Students Learn Mathematics?

Mathematical concepts are abstract. The process of learning mathematics involves internalizing the concepts involved (Cobb, Yackel, & Wood, 1992; Puchner, Taylor, O'Donnell, & Fick, 2008). As students learn, they need representations of these concepts before they can internalize them and work with them abstractly (Beattie, 1986). In other words, they need to be able to relate the concepts to parts of their own world that they have experienced (Cain-Caston, 1996). Manipulatives are a source for these "world experiences".

According to Bruner (1960), the use of concrete activities that progress to abstract concepts allows students to understand the reversibility of mathematical operations. Also, the act of learning involves three processes, the acquisition of new information, the transformation or analysis of the information, and the evaluation of the new knowledge. The use of manipulatives allows students to naturally go through this learning process.

Dienes (1960) uses Piaget's stages of learning to explore the process of learning and the use of manipulatives. Piaget's stages of learning cover a child's development from play to purposeful play to understanding and practice. Dienes emphasizes the need for varied experiences and variables when practicing a concept. The use of manipulatives

allows students to progress through the natural stages of learning and help build the mathematical structure in their minds. The teacher's role is to keep communication open and to construct activities that bring students through the learning stages. Dienes gives four principles to follow. First is the dynamic principle, which involves activities that provide experiences that mathematical concepts can be built, it is a time for students to play and become familiar with the materials. Second is the constructivity principle, which suggests teachers allow students to work with the materials before they analyze the concepts, in other words, have students work through structured activities before declaring the concept itself. Third is the mathematical variability principle, which states that students should be presented with a variety of variables within a situation. Fourth is the perceptual variability principle, which states that a variety of examples or situations should be presented to students to enforce a concept.

Manipulatives have the ability to display mathematical concepts. The important part is not the object themselves, but their interaction with each other as they are moved and manipulated. This allows the students to create their own mental pictures and concepts (Keller, 1993; Boulton-Lewis, 1998; Meira, 1998). The actions on the manipulatives and the manipulatives themselves create experiences and knowledge for the students that allow them to build new knowledge and create their own mental images. The key is teachers guide the students in their translation between the manipulatives and the mathematical concept being learned (Bright, 1986; Kennedy, 1986; Cobb, Yackel, & Wood, 1992; Moyer, 2001). The theories behind the use of manipulatives believe that the more experiences students have with the concrete representations, the better they will be

in making the translations from their world to the abstract mathematical concepts (Kennedy, 1986).

When learning and remembering, visual processing creates information that is easier to recall than verbal processing (Suh & Moyer, 2007). Manipulatives automatically involve the visual aspect of learning. If they are taught along with conversations about the concepts being learned they become even more powerful as they involve two mental representations, visual and verbal (Suh & Moyer, 2007). Also, the visualization that is connected to the use of manipulatives is beneficial to the higher order thinking skills (Keller, 1993).

Mistakes Made While Teaching with Manipulatives

Manipulatives are a teaching tool that can be very attractive to mathematics teachers. However, they are not a "fool-proof" tool. Many mistakes can be made while using them for instruction. In order to avoid them, it is important to know what mistakes can be made.

One of the most common mistakes written about in the reviewed research is that manipulatives are not transparent, meaning that the mathematical concepts being taught using the manipulatives are not automatically understood and seen by the students (Ball, 1992; Cobb, Yackel, & Wood, 1992; Thompson, 1994; Moyer, 2001; Puchner, Taylor, O'Donnell, & Fick, 2008). The manipulatives are created by people who already know the mathematical concepts the manipulatives were created to teach. Teachers often "see" the concept meant for the manipulatives and assume the students will easily "see" the same thing. However, the students may "see" other concepts in those same manipulatives (Ball, 1992; Moyer, 2001; Puchner, Taylor, O'Donnell, & Fick, 2008).

Because students may "see" other concepts present in the manipulatives, teachers need to become extremely familiar with the manipulatives they are using. They need to be aware of the multiple representations and able to recognize when students are using those instead of the one intended. Many times, teachers assume students are using the intended representation and the communication breaks down (Thompson, 1994).

It also happens that many students do not understand the connection between the physical manipulative and the mathematics concept being taught. Not that they "see" a different representation, but that they do not see any connection at all. Then it becomes just one more thing to learn instead of an aid in understanding the mathematical concepts being taught (Kilpatrick, Swafford, & Findel, 2001). According to Suh & Moyer (2007), the loss of a connection may be due to a cognitive overload when working with the manipulatives and symbols at the same time, students are unable to keep track of everything at one time. It also may be due to teachers not using them effectively. They may not be guiding students to the concepts (Heddens, 1986), or they may not understand the use of that manipulative themselves (Ball, 1992).

Communication is another place teachers make mistakes when teaching with manipulatives. We need to give the students a chance to communicate their understanding and reflect on what they are doing with the manipulatives. This will allow them to formalize their understanding of the concepts they are learning and cue the teacher to any misunderstandings (Heddens, 1986; Moyer, 2001). According to Resnick (1983), students will try to make sense of what they are learning, even without all of the information. As a result, they may come up with misguided, incomplete, and sometimes

incorrect theories. A lack of communication would mean those theories are never corrected.

Another misuse of manipulatives in the mathematics classroom is when they are chosen as a method of calculation instead of a tool for understanding (Kilpatrick, Swafford, Findel, 2001; Kilpatrick & Swafford, 2002; Puchner, Taylor, O'Donnell, & Fick, 2008). Also, according to Ambrose (2002), girls have a tendency to use concrete materials to solve problems and not advance to other methods such as mental math or abstract computations. When selecting manipulatives for the classroom, teachers are all too often looking for them to help students do something instead of have them help the students understand a concept (Thompson, 1994).

Also, teachers may not think carefully enough about how the manipulatives will help students learn the concept (Balka, 1993; Puchner, Taylor, O'Donnell, & Fick, 2008). Studies also show that when teachers use the manipulatives in a prescribed manner, they may be taking away the purpose of using the manipulatives during instruction. Instead of understanding the concept, students are just learning another process, which hurts the learning of the underlying concept being taught. This also leads to disregarding students' alternative methods or uses with the manipulatives that may be legitimate but not the prescribed method or use, which becomes a missed opportunity for valuable communication and a deeper understanding (Thompson, 1994; Puchner, Taylor, O'Donnell, & Fick, 2008).

Another commonly discussed mistake in the research reviewed is the lack of time given to students to work with the manipulatives (Heddens, 1986; Clements & McMillen, 1996; Kilpatrick, Swafford, Findel, 2001). According to Moyer (2001), students need to

be extremely familiar with the manipulatives in order to learn from them effectively and avoid cognitive overload. This can only be done with an adequate amount of time.

The last mistake discussed here is how and why teachers decide to use or not use manipulatives for instruction. At the secondary level, it is common for teachers to not use manipulatives. Unfortunately, students may have difficulty transitioning from learning with manipulatives to learning at an abstract level only (Boulton-Lewis, 1998). There are also times when teachers might use manipulatives as a reward or take away manipulatives as a punishment. This view treats the use of manipulatives as a fun exercise instead of a tool for learning (Moyer, 2001). Lastly, manipulatives do not have to be required for all students; some may do better with paper and pencil or may not need them as long as others (Clements & McMillen, 1996).

The Process of Using Manipulatives in the Classroom

It is important to not dive into using manipulatives without appropriate guidelines. The following research discusses the teacher's job before using manipulatives in the classroom, the process of teaching with manipulatives, the teacher's role during instruction, and the timeframe expected for successful learning.

Before using manipulatives in the classroom, it is important to do preliminary work. When teachers are considering the use of manipulatives, they need to focus on what they want their students to learn and understand and not what they want their students to do (Thompson, 1994). It is necessary that teachers are extremely familiar with the manipulatives they are using for their instruction. They need to be aware of the different uses and interpretations students can find. They need to be ready for the different questions that will arise and be prepared for the different thoughts that students

may have. Teachers also need to make sure the manipulatives correctly model the concept being learned. An added result of this preparation, other than student learning, is improved student attitudes towards mathematics and higher achievement levels (Balka, 1993; Thompson, 1994; Clements & McMillen, 1996; Moyer, 2001; Puchner, Taylor, O'Donnell, & Fick, 2008). In the end, it is important that the teacher commits to their decision to use manipulatives and is willing to analyze the results and make necessary adjustments (Johnson, 1993).

The work of Bruner (1960) and Dienes (1960) introduces and analyzes the process of teaching with manipulatives. This process is, for the most part, agreed to as three stages: concrete, representational, and abstract. Some research includes other stages such as semi-concrete and semi-abstract, but ultimately the process is the same. During the concrete stage, students are working with the manipulatives only. They are exploring the concepts being taught and should be reflecting on and justifying their actions. Once they have an established mental representation of the concept, students move on to the representational stage. Here they can draw pictures, watch a demonstration, use virtual manipulatives, use a number line, etc. They should also be making a connection between their representational actions and the related symbolic manipulations. The last stage is the abstract stage. At this point, students are ready to work only with symbolic manipulations. Emphasis during all the stages should be on the connections amongst the actions on the manipulatives, the meanings of the concepts, and the steps of the algorithms (Beattie, 1986; Bright, 1986; Heddens, 1986; Schultz, 1986; Johnson, 1993; Keller, 1993; Boulton-Lewis, 1998; Moyer, 2001; Kilpatrick, Swafford, & Findel, 2001; Moyer, 2004).

Variations of this process include: allowing mature students a more open-ended experience (Moser, 1986), having a more simultaneous approach where the work with manipulatives is accompanied by an introduction to a recording system and followed by reflection and applying (Bruni & Silverman, 1986), and Witzel and Allsopp (2007) emphasize the need for learning disabled students to verbalize their thoughts by making "Think-alouds" a step in the process. Thornton (1986) also emphasizes the need for learning disabled students to verbalize their thoughts as this will help create internal representations. He suggests teachers demonstrate and talk through each step, then demonstrate and write each step as students verbalize the steps. The next step would be to talk and write, then follow with the manipulatives to check. The last two steps are to help students connect what they learned from what they learned before and to work at the abstract level, checking only once in a while with manipulatives.

While instructing with manipulatives, there are some things to consider. Teachers need to create opportunities and guide students to making the connection between the concrete and abstract stages (Heddens, 1986; Johnson, 1993; Moyer, 2001; Kilpatrick & Swafford, 2002). The activity of using the manipulatives creates the meaning, not the actual object itself (Cobb, Yackel, & Wood, 1992; Meira, 1998). When planning the lessons, teacher should include experiences that help students see conflicts and make corrections (Cobb, Yackel, & Wood, 1992; Clements & McMillen, 1996). Manipulatives are more useful when testing new or expanding ideas rather than after students have already learned a concept (Puchner, Taylor, O'Donnell, & Fick, 2008). Make sure students have enough time to become extremely familiar with the manipulatives, in return they can better reflect and analyze their actions on the manipulatives(Johnson, 1993;

Boulton-Lewis, 1998; Kilpatrick, Swafford, & Findel, 2001; Moyer, 2001). Also, allow students to be creative and to explore the multiple representations that the manipulatives can be; create a classroom where mistakes are allowed and learned from (Thompson, 1994; Ambrose, 2002).

Communication is a key ingredient to the success of manipulatives. Teachers need to listen to students' comments and questions to gauge their understanding and their need for remediation (Thornton, 1986; Ball, 1992; Johnson, 1993). Teachers should encourage and create opportunities for students to verbally reflect, justify their actions, and analyze their mistakes with their peers and teacher; which can help make the connection between the concrete and abstract stages (Bright, 1986; Heddens, 1986; Thornton, 1986; Clements & McMillen, 1996; Boulton-Lewis, 1998; Moyer, 2001). It is also important that teachers ask students meaningful questions that guide students from the concrete to the abstract (Heddens, 1986; Thornton, 1986; Ambrose, 2002). Instead of asking questions that begin with "what", teachers should ask questions that begin with "why" and "how" (Heddens, 1986).

A difficult challenge to teaching with manipulatives is deciding how much time should be given to their use. For the big picture, studies show that the length of time given to the process of teaching with manipulatives is related to achievement, specifically a school year or longer (Sowell, 1989). It is suggested that the manipulatives are used across topics, which keeps students familiar with the specific manipulative (Moser, 1986). At the day-to-day level, the time allotment varies. To begin with, students need to be given time to "get to know" the manipulatives, to get playing out of their system and be familiar with the objects (Moser, 1986; Johnson, 1993; Moyer, 2004). In general,

students need to be given time to think, explore and analyze (Heddens, 1986; Johnson, 1993; Clements & McMillen, 1996; Kilpatrick, Swafford, & Findel, 2001). How much time needs to be determined by the teacher. The high achieving students may not need much time with the manipulatives before they move on to the abstract level, but the learning disabled students may need a much longer amount of time at the concrete stage (Clements & McMillen, 1996).

Chapter 3: Interpretation

The research reviewed covers many topics within the world of using manipulatives in the classroom. First, manipulatives help students learn mathematical topics by giving them a visual connection, a link to prior knowledge, and help create an internal representation of the external, abstract concepts. Second, they provide teachers and students an avenue for communication and insight to the students' thoughts and understanding. They allow students to be active learners, which is especially helpful for learning disabled students. Although many teachers worry about the time requirement, teaching with manipulatives covers as much material as abstract methods. Third, there are many common mistakes teachers make when teaching with manipulatives. Teachers often look at manipulatives as an obvious representation of the mathematics concepts being taught, which is not always the case for students. Also, teachers often do not give the manipulatives adequate time, whether it is teachers taking the time to get familiar enough with them or giving the students enough time to process through the use of them. Last, the research covers the proper process for teaching with manipulatives. One of the main processes is the use of the concrete, representational, and abstract stages. The research also stresses the use of communication to help students make connections between the concrete and abstract stages.

Manipulatives have been a relatively new addition to my classroom. I have used them extensively with my students for two years. What I have seen in my classroom parallels what the research finds. In this chapter I will:

- describe my classroom and school district,
- discuss the benefits I have observed in my classroom,

- discuss the mistakes I have made,
- describe the general process I use when teaching with manipulatives,
- list the manipulatives I have found to be especially useful.

My Classroom

I teach in a small school district, consisting of grades kindergarten through twelfth grade in one building. It is located in a rural area of Northwestern Minnesota and consists of four different communities. Although the communities share a school district, they do not intertwine any other part of their communities.

My school district's student population, out of 500 students, has 12% special education, 48% free and reduced lunch, and 98% white. It is noteworthy that of the majority of white Christian students, there is a minority of approximately 25% Russian Orthodox students. My typical classroom setting has slightly less than twenty-five students, which can vary from as few as 6 students to as many as 34 students. I teach various classes from 7th grade Mathematics to College-level Algebra, Trigonometry and Calculus.

The Benefits of Using Manipulatives in My Classroom

Much of what the research indicates as the benefits of using manipulatives, I have found true in my own classroom. Following, I discuss the various benefits, including how manipulatives allow me to teach at a concrete level, how students become active learners, how manipulatives open the doors of communication, and how manipulatives have allowed me to reach my learning disabled students.

Manipulatives allow me to introduce a concept at a concrete level, a level that intertwines with the students' prior knowledge. On this foundation I can build to the point

of abstract computation, with the flexibility to differentiate to the students' level of understanding. Along the same lines, I have found manipulatives to be a great tool for remediation and tutorial times. They have helped move the student to a greater understanding in less time. As a result of greater understanding, the students don't ask the common questions they used to ask anymore; questions like, "Why is 3 + 2x not equal to 5x?"

The research indicates that manipulatives allow students to be active learners and gives them a visual aid to their understanding, memory and recall. This has been very apparent in my classroom as students who normally are very passive in their learning and participation in class, take an active role with the manipulatives. They physically work with the manipulatives, discuss their actions with their partner or group members, attach the notation to the action, and share their findings with the rest of the class. After working with manipulatives, I have had several students answer questions in class who would otherwise just shrug their shoulders.

The next benefit I have found with the use of manipulatives is conversation. As the research claims, manipulatives give the students and teachers an avenue for conversation and understanding. A struggling student now has an object they can reference in their question instead of not knowing what words to use. In doing so, I can know what level of understanding they have instead of them telling me, "I don't know how to do this."

Above all, the use of manipulatives has enhanced my success with my learning disabled students. Teaching in a small school, I have had many of these students in my class for multiple years. They have shown greater understanding and success in the past

two years than I have seen in the four years prior, the difference is the use of manipulatives. They have become animated in class and are able and willing to discuss the topics with their classmates. I have even seen these students strive to figure the concept out for themselves, instead of having their classmates help them.

The Mistakes I Have Made While Instructing with Manipulatives

As many other teachers, I have made mistakes while using manipulatives in the classroom. One of the biggest topics within the research is that the concepts the manipulatives represent are not transparent to students, no matter how obvious it is to the teacher. This is one of my biggest mistakes. I have not always spent enough time helping the students understand the connection. It has been a frustration for me when the students were confused about something I thought was quite obvious. However, the time needs to be spent getting the students familiar with the manipulatives and understanding the connection between the object and the abstract concept.

Time has also been a big issue for my classroom. I have learned from experience and from the research, the time is worth spending on manipulatives. The students need the time to build the connections and in the end that time is saved with less remediation. I have also found that my advanced students do not need the same time as the other students. In fact, many of my advanced students dislike using manipulatives and would rather jump straight to the abstract level. The research shows that advanced students benefit from the use of manipulatives also, but the time needed for them can be significantly less.

As the research states, the best time to use manipulatives is when introducing concepts instead of after the students have already learned them. I found this to be true

when I visited a second grade classroom to introduce the use of base ten blocks when teaching addition with renaming. The students had already learned the concept and I found it difficult to help them make the connection because they kept jumping to the answer they had already figured out abstractly. The teacher and I agreed that the base ten blocks would be extremely useful at the beginning of the year to introduce the concept, but were not the most effective at the time of year I introduced them.

The last mistake discussed here is one I still have to fix. I have found it difficult to use virtual manipulatives in class. The lack of computer availability and the lack of time have been my biggest obstacles. However, virtual manipulatives can be a useful tool as they help students make the connections between the concrete and abstract. They also may be more appealing to the students than the physical object, which they may see as more juvenile.

The Process for Teaching Manipulatives in My Classroom

There are many aspects to consider when discussing the process of teaching with manipulatives. The research is very clear that any process used must help make a connection between the manipulative and the abstract concept being taught. To begin with, the manipulatives need to be introduced and the students need time with the manipulatives to become familiar and the get the "play" out of the way. If possible and appropriate, the manipulatives should be used year round. Research shows the longer the students use the manipulatives, the more they achieve; it also keeps the students using an object that is familiar to them with less time needed to introduce it.

The use of the concrete, representational, and abstract stages are also extremely important. For the concrete stage, students work only with the manipulatives. I introduce

the concept we are learning and then give the students problems to work on and discover the mathematics to be learnt. I spend a lot of time with the representational stage. I often have the students work in partners, one working with the manipulatives and one writing out the abstract symbols and procedures. This way, much like virtual manipulatives, the students can simultaneously work with the manipulatives and see the abstract symbolic representation; it also gives them the opportunity for valuable communication. I will also give homework in which the students diagram the steps they would have taken with the manipulatives and the abstract work that goes with each step. The final outcome is the abstract stage. This is where they get their homework and assessments. Often, during the entire process, we will stop as a class to share and discuss what they are finding to be true and difficult.

There are times during instruction in which we are working at an abstract level and students make comments that demonstrate their misunderstandings. At this point, I will often bring out the manipulatives to help them understand what they are missing. I like this because it brings the class back to a common ground in which they comfortable and are able to maneuver.

The Manipulatives I Have Found Useful in My Classroom

There are specific manipulatives I have found to be extremely useful and versatile in my middle and secondary classrooms. I hope to discover more manipulatives as I continue teaching. There are manipulatives I have not yet explored; these include Cuisenaire rods, Tangrams, pattern blocks, pan balances, and more. Below are the descriptions of the manipulatives I have used in my classroom.

Algeblocks. One of the most commonly used manipulatives in my classroom is Algeblocks. They are a manipulative designed to model polynomials and the actions that can be performed on them. They can also be used for integer operations. I have used them in my seventh grade classroom up to my Algebra II classroom. I have also recommended them to many upper elementary teachers. Algeblocks allow me to use a geometric approach to Algebraic concepts. For example, I often reference the use of arrays when students were first learning about multiplication to the multiplication and division of polynomials. Algeblocks have taken very abstract concepts, such as solving equations, simplifying expressions, solving systems of equations, and composition and brought them to a concrete level. I have also used them in the middle school to help students understand integer operations (also resulting in rational number operations), a concept that I have struggled teaching in the past, but now have great success based on observations during class.

Base-ten blocks. Although I have not used base-ten blocks in the classes I have taught, I have recommended them to the elementary teachers and have seen great results. The base-ten blocks help with concepts such as place value, addition, subtraction, multiplication and division. They also help with concepts of fractions and decimals. Also, using base-ten blocks in the elementary lends well to a connection with the use of Algeblocks in the middle and high school classrooms.

X-Y coordinate pegboards. I once considered graphing to be a concrete procedure. However, looking at the definitions, graphing fits more in the representational stage. X-Y Coordinate Pegboards are plastic boards with 15x15 grid holes. Each board has movable axes. There are two sets of colored pegs, red and blue, and many different rubber bands.

The boards allow students to physically put the points (pegs) in the holes and connect the points with rubber bands. I have used these pegboards to help students understand slope, and to introduce graphing linear and quadratic functions. Some of the upper elementary teachers have also used the boards to introduce coordinate graphing.

Another physical representation of graphing is constructing a life-size coordinate system in which the students act as the points. This can be used to introduce various functions, much like the pegboards. For example, with the students moving to their positions simultaneously as the points, they can visually see how the function develops and why asymptotes may exist.

Linking cubes. Linking cubes have many possibilities. They are cubes that have the capability to connect to each other. I have found these to be very useful when introducing volume and surface area of rectangular prisms. They help students understand the formulas involved, instead of just memorizing them. Linking cubes can also be used in the same fashion as base-ten blocks, except the students have to build the ten's and hundred's, which can be very useful.

Chapter 4: Conclusion

The research on the use of manipulatives in the mathematics classroom is extensive. However, there is still much more to research. Below I attempt to answer my research questions with the research I have reviewed. I also discuss what still needs to be researched and my plans for the use of the research I have done.

How Do Manipulatives Help Students Learn Mathematical Concepts?

1. Is there a part of the learning process that manipulatives help where other methods might not?

Manipulatives allow students to work at a concrete level and progress through a natural learning process to an abstract level. They also expose students to both verbal and visual mental representations, which allows for better understanding.

2. What is the educational psychology behind using manipulatives?

Using manipulatives allows students to go through the natural learning process of working at a concrete level and progressing to an abstract level. With proper activities and teacher guidance, students can use the concrete activities to create new knowledge and build on previous knowledge. The concepts developed from the concrete activities become the foundation upon which to build the abstract concepts. One of the key factors is teacher guidance through the process with communication and appropriate activities.

Also, the use of manipulatives gives students an understanding of the mathematical concepts that allows them to understand the reversibility of the mathematical operations. This becomes very useful when students are learning the mathematical operations and also when they are taking Algebra courses in high school.

Is the Effect of the Use of Manipulatives the Same for All Students?

1. Are manipulatives useful for learning disabled students?

The reviewed research strongly encourages the use of manipulatives with learning disabled students. Manipulatives give students a multi-sensory learning experience that allows them to be active learners. They also allow for more accessible conversation with the physical manipulatives at the center instead of the abstract concepts. In my experience, manipulatives have been the most successful with this group of students. They have become more involved in the class activities and conversations, and have been more academically successful.

2. How do gifted and high achieving students do with manipulatives?

The reviewed research suggests that the high-achieving students benefit from the use of manipulatives and so they should still use manipulatives; however, they may not need the same length of time at the concrete-stage as the rest of the students. In my experience, these students have complained that manipulatives make things more confusing and would rather jump to the abstract-stage of instruction. I have also found this to be the case for myself. But when I finally took the time to explore manipulatives, I found that I understood concepts at a whole new level. The challenge to these students is to give the manipulatives a chance and get a deeper understanding.

What are Mistakes to Stay Away From When Using Manipulatives?

There are many mistakes to stay away from when using manipulatives in instruction. To begin with, many teachers see the mathematical concepts present in the manipulatives very easily. However, students may not see these concepts as readily.

Teachers need to take the time to make sure students make the connections between the physical objects and the mathematical concepts.

Another mistake teachers make is not allowing for enough communication. Giving the students an opportunity to communicate with each other and the teacher gives them the chance to work through the learning process and also allows the teacher a chance to discover what the students understand and to address any misunderstandings.

A common misuse of manipulatives is to use them as a calculation tool instead of a tool for understanding. This is closely linked to teachers not taking the time to understand how the manipulatives will help students learn and how the manipulatives correctly represent the mathematical concept. Teachers need to take time when designing the lesson to closely analyze how manipulatives will help students learn the intended concept and to understand all the different confusions and representations that can happen during instruction.

Lastly, a common mistake, one of which I am guilty, is not giving students enough time to process through the use of manipulatives. Thinking and learning takes time, students need to be given enough time to get familiar with the manipulatives being used and to make the connections to the abstract concepts. This will be a different amount of time for each student, which can be difficult when teaching a large group of students.

What Process or Methods Should Be Used When Teaching with Manipulatives?

A common process to follow is the concrete-stage, representational-stage, and abstract-stage. Students begin with the concrete-stage in which they move the manipulatives around, becoming familiar with their properties and the properties they represent in the mathematics concepts. Then they move to the representational-stage

where they draw the manipulative instead of working with the manipulative itself. It can also be where students use pictures or watch someone else move the manipulatives. The abstract-stage is the point where the students are working exclusively with the symbols involved with the mathematical concept.

The importance of this process is that the students are assisted with making connections between the concrete- and abstract-stages. An added step during the representational-stage is to have students use the physical manipulatives and write the abstract steps at the same time. This can be done well with partners. The use of virtual manipulatives allows students to see both the manipulatives and the abstract steps at the same time.

1. What makes this process/method effective?

Students' natural learning goes through a process of discovering the new concept, analyzing and exploring the new concept, and then evaluating the truth of the new concept. The process of teaching with manipulatives brings students through this process by giving them the opportunity and means to explore the concepts on a concrete level and progressively to an abstract level. They also provide students the experiences to connect their previous knowledge to new knowledge. By combining the concrete and abstract during the representational level, it allows students to self-check their methods and understanding. By communicating and summarizing, the students have a chance to evaluate the correctness of their new information and knowledge.

Manipulatives also allow teachers to meet students at their developmental level. Classrooms are full of students who have had various life experiences. Some may not

have yet had the concrete experiences necessary to understand the abstract concepts present in a mathematics classroom.

2. When is it appropriate to use manipulatives?

Manipulatives are appropriate to use whenever there is a manipulative that represents the mathematical concept well. It is important for the teacher to work with the manipulatives before instruction to ensure that the manipulative is appropriate for the concepts being taught.

It is suggested that manipulatives are used to introduce new concepts rather than after the concepts have already been taught. However, manipulatives work well as tools for remediation.

3. Is there a specific time frame for each step in this process?

The reviewed research does not give a specific time frame for each step in the process. It does suggest that the longer manipulatives are used, the more success the students will have. More specifically, a year's timeframe has had the best student achievement. Ultimately, teachers need to allow students enough time to become familiar with the manipulatives and to make the connections between the concrete- and abstract-stages.

A Call for More Research

An important part of using manipulatives in mathematics instruction is proper professional development for the teachers. At what part of a teacher's experience is the most effective time to pursue the use of manipulatives? My experience was that at the beginning of my teaching career I had much to learn and improve upon that the use of manipulatives was not in my sight. It took a few years for me to get settled enough to

start exploring the methods of teaching enough to discover the benefits of using manipulatives. I look back at my notes from my undergraduate methods courses and realize we did cover the use of manipulatives, but it seems that without teaching experiences to reflect upon, I did not retain the lessons.

Also, the psychology of the use of manipulatives is scarce, especially at the high school level. Most research is based at the elementary level; however, manipulatives are very valuable tools in the high school. Is their use the same for teenagers? Is the teenage brain going to utilize the concrete activities the same as a child's brain? Is the process to the abstract the same for high school students?

How Do I Plan on Using This Research?

This research has been extremely valuable to my career. In my experimenting with manipulatives in my classroom, I have found that there was much more that I needed to learn to maximize student learning. My research has given me much to consider as I plan for the up coming year. One of the biggest concerns I have is the time frame I will be giving the use of manipulatives for certain concepts. Last year I found that the time I gave to the manipulatives actually saved me time later in remediation. I plan to allow for even more time this coming year, especially for my learning disabled students.

In my school district, I have taken on the role of math specialist for grades kindergarten through 12. I have visited often with teachers, helping brainstorm ideas for the mathematical concepts being taught. Through this process we have come up with many unique ideas for instruction, including use of manipulatives. I hope to continue this role and use what I have learned from the research to continue to come up with more ideas.

As a participant of the Math and Science Teacher Academy, I will be conducting Professional Learning Communities within my school district for grades three through eight once a month. I plan to use some of those meetings to share my research with my fellow teachers and help them explore the manipulatives available for those grade levels.

I have also been asked to present at a workshop for our region by the Northwest Service Co-op in January. The main concept for the session is the use of manipulatives. I hope to share my research from this paper and also share the manipulatives I have found useful in my classroom.

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Ambrose, R. C. (2002). Are we overemphasizing manipulatives in the primary grades to the detriment of girls? *Teaching Children Mathematics, 9*(1), 16-21.
This article claims that girls tend to stay with the use of manipulatives and have difficulty moving to more abstract methods. It also discusses how students move from one problem solving level to another. It then gives suggestions for how to get students to use mental math but not necessarily take away the manipulatives. The article will be useful for the special take it has on helping students move to mental math methods of problem solving.

Balka, D. S. (1993). Making the connections in mathematics via manipulatives. *Contemporary Education*, *65*(1), 19-23.
This article gives examples of appropriate uses of manipulatives and discusses the reasons for the use of manipulatives. It does not go into great depth, but it is a good resource for ideas and reasons for manipulatives.

Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of math education. *American Educator, 16*, 14-18, 46-47.
This article warns of the assumption that manipulatives are automatically a representation of the mathematical concepts being taught. It emphasizes the need to listen to what the students are saying in order to understand what they see the manipulatives as representing. I find this article to be extremely useful in my research. It addresses the pitfalls of using manipulatives, but also how to approach their use correctly. It emphasizes that they are not a fool proof method of teaching and learning.

Beattie, I. D. (1986). Modeling operations and algorithms. *The Arithmetic Teacher*, *33*(6), 23-28.
This article addresses the reason for teach algorithms considering the available technology. It also considers the way in which algorithms are taught, citing that going straight to abstract means that you could be bypassing the complete understanding of the algorithm. The article is a good addition to my research because it addresses how to use the manipulatives and also discusses the purpose of the manipulatives.

Boulton-Lewis, G. (1998). Children's strategy use and interpretations of mathematical representations. *Journal of Mathematical Behavior, 17*(2), 219-237.
This article looks at learning theory and that the use of manipulatives while learning given algorithms may be too much going on for a student's brain. It also talks about the gap between the physical use of the material and the abstract concepts and suggests the appropriate use of manipulatives. The article will be useful for my research; it gives theory behind the use of manipulatives and suggestions on how to use them.

Bright, G. W. (1986). Using manipulatives. *The Arithmetic Teacher, 33*(6), 4.
This article warns of the danger that students may not see the connection between the concrete manipulatives and the symbolic notations. It suggests that oral language can help with this connection. This will be a useful article as it criticizes what can go wrong with manipulatives and also gives suggestions for success.

Bruner, J. S. (1960). *The process of education.* London, England: Oxford University Press.
This book analyzes the way students learn and think. It give much insight to how students learn at certain ages and what teachers can do to help prepare them for future ages. It is very useful for my research in the aspect of the process students learn and how manipulatives will help. It did not specifically discuss the use of manipulatives as much as I had hoped, but there are inferences that can be made from its content.

Bruni, J. V., & Silverman, H. J. (1986). Developing concepts in probability and statistics—and much more. *The Arithmetic Teacher, 33*(6), 34-37.
The article focuses on specific lessons for probability and statistics. There is very little discussion about the use of manipulatives in general. It does, however, give a four-step method for using manipulatives in probability lessons, but it can be used in other areas also. It may be useful for my research in that it supports other articles.

Cain-Caston, M. (1996). Manipulative queen. *Journal of Instructional Psychology*, *23*, 270-274.
This study was based in the third grade level, comparing the use of manipulatives to no use of manipulatives. The study found in favor of the manipulatives, citing that it allows students to relate math to the physical experiences with the manipulatives. I will find this study useful in its additional support of the use of manipulatives.

Clements, D. H., & McMillen, S. (1996). Rethinking "concrete" manipulatives. *Teaching Children Mathematics*, *2*, 270-279. Retrieved July 22, 2008, from Wilson Web database.
This article is a collection of suggestions for using manipulatives in the classroom effectively. It has good advice that also looks at the misuse of manipulatives, such as not allowing student enough time to work with the manipulatives and making all students use them when some may do better with paper and pencil. It has direct correlation with my research and will be of great use.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education, 23*(1), 2-33.
This article analyzes the current views of teaching mathematics using physical representations. It argues that students may not interpret the physical manipulatives in the way the teacher does, and thus not correctly the desired concept. It emphasizes the importance of the teacher's role and the social interactions during this learning process. The article is useful for my research because it acknowledges the mistakes that can be made when using manipulatives during instruction, but also addresses what can be done differently.

Dienes, Z. P. (1960). A theory of mathematics-learning. In *Building up mathematics* (pp. 19-36). London: Hutchinson Educational.
This book is an overview of the teaching of mathematics in many different areas. The second chapter is specifically about the learning process that students go through. It is a useful chapter for my research as it covers the learning process in general, specifically in the mathematics area.

Dorward, J. (2002). Intuition and research: Are they compatible? *Teaching Children Mathematics*, *8*(6), 329-332.
This article analyzes the use of virtual manipulatives. It discusses the advantages of virtual versus concrete manipulatives. It also follows research of three teachers using various levels of manipulatives. This is an important look at another forms of manipulatives. I find it very useful in my research.

Heddens, J. W. (1986). Bridging the gap between the concrete and the abstract. *The Arithmetic Teacher, 33*(6), 14-17.
As the title suggests, this article gives suggests of how to bring students from understanding the manipulatives to understanding the paper and pencil algorithms. It stresses the importance of giving the students enough time and opportunity for conversation. The article will be extremely useful for my research in that it directly addresses my research questions.

Johnson, K. A. (1993). Manipulatives allow everyone to learn mathematics. *Contemporary Education*, *65*(1), 10-11.
The article contains a list of suggestions for teachers when using manipulatives in their lessons. It is a good, concise list that touches on many of the topics discussed in other articles. I will find it very useful in my research because of its organization of common themes.

Jones, S. (2000). *The role of manipulatives in introducing and developing mathematical concepts in elementary and middle grades*. Retrieved on June 12, 2009, from http://www.resourceroom.net/math/Jones_mathmanip.asp
This article enforces that manipulatives should be used for understanding concept and not for computation. It also walks through the process of teaching with manipulatives, including when to diverge from their use. This is a very useful article for my research, it addresses many of the concepts and concerns I have been looking for.

Kaput, J. (1989). Linking representations in the symbol system of algebra. In C. Kieran & S. Wagner (Eds.), *A research agenda for the learning and teaching of algebra* (pp 167-194). Hillsdale, NJ: Lawrence Erlbaum.
This article addresses the psychology behind learning math concepts. It focuses on the types of notation in math concepts and how those notations are worked with. Although it is a good article for the topic of five representations, there is not much correlation with the use of manipulatives.

Keller, J. D. (1993). Go figure! The need for manipulatives in problem solving. *Contemporary Education*, *65*, 12-15.
This article talks of how students lack the physical experiences of old because of the technology that exists today. It also walks through the levels of development or learning, starting with the concrete stage. It emphasizes that the use of physical experiences help in the higher levels of learning. This article will be useful in my research as it goes through the levels of learning and helps argue for the use of manipulatives, especially as students have fewer and fewer concrete experiences on their own.

Kennedy, L. M. (1986). A rationale. *The Arithmetic Teacher, 33*(6), 6-7, 32.
This article explores the learning theory use of manipulatives. It suggests that the combination of all of the research suggests the use of manipulatives is helpful for students. It also suggests the more variety and longer time of use, which includes all grade levels, is even more beneficial. This will be a useful article for my research in analyzing the usefulness of manipulatives in the learning process.

Kilpatrick, J. & Swafford, J. (Eds.) (2002). *Helping children learn mathematics.* Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. National Research Council. Washington, DC: National Academy Press.
This book discusses the process students learn through and how teachers should approach the lessons they teach. It addresses the use of manipulatives and how they can assist in the process of learning, but not used as a solution method. The book will be very useful for my research. It is a holistic view of how students learn math, especially with the use of manipulatives.

Kilpatrick, J., Swafford, J., & Findel, G. (Eds). (2001). Teaching for mathematical proficiency. In *Adding it up: Helping children learn mathematics* (pp. 313-368). Retrieved from National Academies Press database.
This section of the book focuses on the use of manipulatives. It warns that manipulatives are a useful tool in that it helps students learn concepts, gives opportunity and means for conversation, and allows students to self-correct, but they are not the only method that should be used to teach. This will be useful for my research. It talks of appropriate use of manipulatives and also warns of misuse.

Lemke, M., & Gonzales, P. (2006). *Findings from the condition of education 2006: U.S. student and adult performance on international assessments of educational achievement.* Retrieved August 6, 2008, from http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2006073
This publication analyzes the results of a few international assessments and how the students in the United States perform. For my research, this helps with my justification for why we need to look at other methods of research, specifically using manipulatives.

Meira, L. (1998). Making sense of instructional devices: the emergence of transparency in mathematical activity. *Journal for Research in Mathematics Education, 29*(2), 121-142.
The article explores the transparency of objects being used in mathematics instruction. The article explores a small experiment teaching linearity using different devices. It was not the most convincing article. From my other research, it almost seems like the point of instructional devices was lost in the research. I don't know that I will be using it in my research.

Moser, J. M. (1986). Curricular issues. *The Arithmetic Teacher, 33*(6), 8-10.
This is a short article that gives an overview of the use of manipulatives. It does discuss the use of manipulatives with advanced students and the long-term use of manipulatives, which adds to the research I have already collected.

Moyer, P. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47*, 175-197. Retrieved July 22, 2008, from Wilson Web database.
This article followed ten teachers and their use of manipulatives throughout the year. It emphasizes that manipulatives do not automatically carry the mathematical concepts teachers may be intending. It discusses the need for making the connections for the students. I will find this research/article to be very useful, as it is a reminder and warning of the misuse of manipulatives in the classroom and will help in addressing their appropriate use.

Moyer, P. (2004). Controlling choice: Teachers, students, and manipulatives in mathematics classrooms. *School Science and Mathematics.* Retrieved June 12, 2009, from http://findarticles.com/p/articles/mi_qa3667/is_200401/ai_n9348825/
The article is a study of ten teachers over a year's time. It demonstrated that giving the students manipulatives freely throughout the class, it allows them a non-public way to get help, without interrupting class. The article will be useful in that it gave another view for the use of manipulatives.

Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives? *Teaching Chidren Mathematics*, *8*(6), 372-377.
This article focuses on the different types of virtual manipulatives and their benefits. As technology becomes more and more available, virtual manipulatives must be a part of our curriculum and thus a part of my research of manipulatives. The article will be useful in my exploration of the virtual side of manipulatives.

Puchner, L., Taylor, A., O'Donnell, B., & Fick, K. (2008). Teacher learning and mathematics manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School Science and Mathematics*, *108*(7), 313-325. Retrieved January 15, 2009, from Wilson Web database.
This article is a collective case study of the use of manipulatives in the K-8 classroom. It emphasizes that manipulatives are not an end to a means, but rather a tool to help create an internal representation. It warns of common mistakes made by teachers, and makes suggestion to help teachers better understand and focus their use of manipulatives. This is a very valuable article that addresses many of my questions in my research.

Raphael, D. (1989). The influence of instructional aids on mathematics achievement. *Journal for Research in Mathematics Education*, *20*, 173-190.
This article analyzed data from the Second International Mathematics Study and over one hundred Ontario teachers and their students. It emphasized that the use of manipulatives was related to a higher coverage of material. I found this article to be in comparison to my own experiences. Although it does not address how to use manipulatives, it does help with the argument for the use of manipulatives.

Resnick, L. B. (1983). Mathematics and science learning: A new conception. *Science, 220*, 477-478.
This article discusses the process of learning and how students connect new concepts to previous knowledge, and that knowledge is clustered together. This topic is important in the argument for the use of manipulatives and will be useful for my research.

Schultz, K. A. (1986). Representational models from the learners' perspective. *The Arithmetic Teacher, 33*(6), 52-55.
This article discusses three learning behaviors very similar to the concrete-representation-abstract learning. It also talks about the types of manipulatives, their actual physical makeup and their nature of use. The article will be use as it gives suggestions of how to use manipulatives and discusses the importance of how involved the students are.

Sowell, E. J. (1989). Effects of manipulative materials in mathematics instruction. *Journal for Research in Mathematics Education*, *20*, 498-505.
This article is an analysis of 60 studies on the use of manipulatives over a long term. The analysis found that concrete manipulatives helped increase achievement. An emphasis was put on the length of the use, a year or more being preferable. This article is a very valuable piece of research. It addresses the length that manipulatives should be used, and gives evidence of the benefit for using it.

Suh, J., & Moyer, P. S. (2007). Developing students' representational fluency using virtual and physical algebra balances. *The Journal of Computers in Mathematics and Science Teaching*, *26*(2), 155-173.
The article explores a project where third grade students learned using physical manipulatives and others learned using virtual manipulatives. It explains the pros and cons for both types of manipulatives. This will be a useful article as it looks at both physical and virtual manipulatives. With the technology available, it is important to utilize the virtual manipulatives and understand their use.

Suydam, M. N. (1986). Research report: Manipulative materials and achievement. *The Arithmetic Teacher, 33*(6), 10, 32.
This is an analysis of research studies in the elementary grades. It found a difference between students who used manipulatives and those who did not. It showed higher retention rates and improvement of attitudes. I find this report to be a substantial claim to the usefulness of manipulatives and it will be useful in my research.

Thompson, P. (1994). Concrete materials and teaching for mathematical understanding. *The Arithmetic Teacher, 41*(9), 556-558.
This article emphasizes that manipulatives are not transparent and that teachers need to be aware of all the interpretations that students can make when working with the manipulative they are using. It states that manipulatives are useful because it gives an avenue for conversation and if gives students something they can act on. This article is useful for my research in that it discusses the correct use of manipulatives, and warns against how manipulatives can be misused.

Thompson, P. W. (1992). Notations, conventions, and constraints: Contributions to effective uses of concrete materials in elementary mathematics. *Journal for Research in Mathematics Educations, 23*, 123-147.
This is a study of twenty, fourth graders who were split between using base ten blocks and a computer program called microworld. It also discusses the practice of having students develop the mathematical notation themselves. Although it would not be a frequent method of teaching, it can help students better understand the reason for the notation. The article was useful in its discussion of the connection students need to make between the blocks and the notation.

Thornton, C. A. (1986). Special learners. *The Arithmetic Teacher*, *33*(6), 38-41.
This article focuses on the learning styles of the learning disabled and the helpful process of teaching with manipulatives. It also looks at the use of manipulatives with the mathematically gifted students. Lastly, it gives a list of ideas for uses of specific manipulatives and useful questions that can be asked. This is a valuable article for my research. It discusses learning processes and gives suggestions for the appropriate use of manipulatives.

Threadgill-Sowder, J. S. (1980). Manipulatives versus symbolic approaches to teaching logical connectives in junior high school: An aptitude x treatment. *Journal for Research in Mathematics Education, 11*, 367-374.
This study explores the use manipulatives and more traditional teaching methods with high and low achieving students. It concludes that the manipulatives were more beneficial for the low achieving students but distracting for the high achieving students. The study will be very useful in my research as it addresses who was benefited by the use of manipulatives and matches much of what I have experienced in my classroom.

Williams, D. E. (1986). Activities for algebra. *The Arithmetic Teacher, 33*(6), 42-47.
This article emphasizes that manipulatives are as useful in Algebra as it is when the students are in elementary. It shares many different Algebra activities. It is not a fundamental part of my research, but it does serves as extra support to the research I have found.

Witzel, B. S., & Allsopp, D. (2007). Dynamic concrete instruction in an inclusive classroom. *Mathematics Teaching in the Middle School, 13*(4), 244-248.
This article discusses the use of manipulatives in the classroom, especially for learning disabled students. It also emphasizes the use of think-alouds to help enhance the use of manipulatives and learning math concepts. The article will be useful for my research as it addresses learning disabled students, a definite part a class's population and needs to be addressed.