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# PROBLEM SOLVING BASED INSTRUCTION IN THE HIGH SCHOOL MATHEMATICS CLASSROOM

by

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## STATEMENT BY THE AUTHOR



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PROBLEM SOLVING BASED INSTRUCTION IN THE HIGH SCHOOL MATHEMATICS CLASSROOM

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This paper is a review of research pertaining to the use of problem solving in classroom instruction. It covers the stages of problems solving as listed by George Polya and how they can be used to enhance mathematics instruction.
This research paper will identify that problem solving is a need based on Minnesota state mathematics standards and National mathematics standards. Covered in this paper is the research on the effects of mathematics instruction using problem solving. The research covers some of the benefits to student education if students learn mathematics using a problem solving based approach to classroom instruction.

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TABLE OF CONTENTS

# Chapter 1: Introduction

Students often struggle in learning mathematics, and much research has taken place on best practices for teaching mathematics. Studies have been conducted on the various aspects that could affect how students learn, in hopes that better practices will be discovered so student learning can be enhanced. One could explore the effects of proper classroom discipline, voice inflection, amount of homework, parental support, etc. In the discipline of mathematics, the use of problem solving skills has been extremely important and highly influential. Problem solving is the foundation of all mathematical and scientific discoveries. Though a few studies have been done on the effects of problem solving, the author feels there is a need to look at the effects of using a problem solving approach in the classroom.

Therefore, the author plans to define problem solving and give examples of that approach of learning mathematics. Then, he will examine the research that has been conducted on the practice of utilizing problem-solving based instruction in the mathematics classroom including its effectiveness and specific strategies that ensure its successful implementation.

## *Statement of the Problem*

While United States educators have made many advances in the past, in today's traditional classrooms, students are often memorizing formulas and practicing routine mathematical procedures. It seems that not enough students are developing the proper problem-solving skills needed to be successful in mathematics. Other countries are surpassing the United States in math by having a higher percentage of students pursue careers in the field of mathematics. According to the TIMSS report (2007), the United

States' national and state test scores, in mathematics, are also lower than various Asian countries including Singapore (Gonzales, Williams, Jocelyn, Roey, Kastberg, & Brenwald, 2009).

Through my observations, this research matches my classroom, a number of students do not enjoy learning mathematics nor do they like being in their mathematics class on a day-to-day basis. Also, many students with great mathematics potential are possibly being classified as needing remedial mathematics services because of their inability to understand mathematical concepts through traditional methods of teaching. Consequently, these students do not usually pursue additional coursework or a career in mathematics, which may potentially decrease the number of quality mathematicians the United States produces.

*Research Questions*

- What are the effects of using problem-solving based learning?
  - What are the strategies for teaching problem-solving based instruction?
  - Can this reduce enrollment in remedial mathematics courses?
  - Will student attitude towards mathematics improve?
  - Will tests scores improve?
  - Can the state standards be met using problem-solving instruction?
- Does a problem-solving approach affect all students in the same way?
  - Is a problem-solving approach beneficial for mathematics students who do not learn well in a traditional setting?

*Significance of the Research Problem*

Educators and parents want to keep more students in the mainstream classroom and less receiving remedial mathematics courses, especially if it is unnecessary. Perhaps the perceived inability to learn mathematics, for students receiving remedial mathematics assistance, is actually math being taught in a way that these students cannot understand. According to *Research Connections in Special Education*, "The NCTM Standards focus on conceptual understanding and problem solving rather than procedural knowledge or rule-driven computation" (Cawley, et al., 2002, p. 1).

According to the NCTM in 1989, "Problem solving is not a distinct topic but a process that should permeate the entire program and provide context in which concepts and skills can be learned (NCTM, 1989, p. 23)" (as cited by Huinker, 1998, p. 170). Gagnon also commented in this article. "Many students with mild disabilities experience difficulties with the math curriculum, but with the right support, they can succeed in a higher level math curriculum" (Cawley, et al., 2002).

As we can see in the previous quotes, there is significance in researching the need for problem solving in the mathematics classroom. Higher state and national mathematics scores are goals. We want students to enjoy learning mathematics and this will lead to overall student improvement in mathematics. According to the Bureau of Labor Statistics "The rise of the Internet has made it relatively easy for part of the engineering work previously done by engineers in this country to be done by engineers in other countries, a factor that will tend to hold down employment growth. Even so, there will always be a need for onsite engineers to interact with other employees and clients" (Engineer Guide USA, 2009). There is a need for students to learn problem solving well,

which is a very important life skill and is necessary in future professions. Mathematics needs to be promoted, but many students choose not to continue their mathematics education in high school, sometimes making it difficult for upper level mathematics courses to be offered in small schools. Working with smaller class sizes may also hinder the ability to include various projects that could enhance the course. Problem solving based teaching is needed to meet demands of society.

*Limitations and Assumptions*

I am limiting my research by referring solely to K-12 mathematics education. The author is using past and current research on problem solving as well as focusing on current ways to effectively teach problem solving.

*Definition of Terms*

**MCTM** – Minnesota Council of Teachers of Mathematics

**NCTM** – National Council of Teachers of Mathematics

**Problem Solving Instruction** – Instruction based on a systematic method of solving problems, both real world and theoretical.

**TPI**-Traditional Problem Instruction, A mathematics teaching technique that uses examples to solve problems then students are assigned numerous problems to work on.

**EAI** – Enhanced Anchored Instruction, A method of instruction that uses real world problems for students to solve to teach mathematics.

**TBI**-Text Based Instruction, Instruction based on text book examples and not on hands on learning.

**LD**-Learning Disabled

**TIMSS**- Trends in International Mathematics and Science Study is an international assessment of mathematics and science knowledge of students around the world.

**Square ONE TV** – A television program aimed at teaching children to problem solve using real world examples.

**DIA**– An approach of instructing mathematics that uses the method of *dia*gnosing errors to help students learn.

**IMP**- An approach of instructing mathematic that uses a method to teach students to *imp*rove by self questioning.

# Chapter 2: Review of the Literature

## *What is problem solving?*

George Polya's work on problem solving has been of great importance in the field of Mathematics. Describing the importance of Polya's findings, the article "Polya, Problem Solving, and Education" in Mathematics Magazine, stated, "For mathematics education and the world of problem solving it marked a line of demarcation between two eras, problem solving before and after Polya" (Schoenfeld, 1987, p. 283). According to George Polya, problem solving consists of four steps: understanding the problem, devising a plan, carrying out the plan, and looking back (Polya, 1945). The National Council of Teachers of Mathematics (NCTM) has written those into similar steps: "Launch; seek out information; explore, experiment and apply; and summarize."(McGehee, 2001, p. 382) The author will explain each of Polya's steps below.

Polya's first stage of understanding the problem starts with a launch. The problems chosen for the student need to be interesting and developed in a way that the student is able to solve the problem at hand. Students need to be engaged and possess the skills needed to solve the problem. In order for students to begin devising a plan they need to fully understand the situation they are working with. Making a problem too hard might discourage a student and cause them to lose interest in finding a solution. A teacher should take special care not to allow students to just make lucky guesses but make sure students understand the problem to be solved (Polya, 1945).

In the second stage, students develop a plan and therefore make connections with their previous knowledge. Coming up with a similar problem from their past, simplifying

the problem at hand to some smaller scale of the big problem, or even experimentation are a few strategies used in developing a plan (Polya, 1945). "If students are to truly believe that mathematics, and fractions in particular, make sense, instruction must allow students to invent their own ways to operate on fractions rather than memorizing and practicing the procedures imposed by the teacher or textbook. In fact, premature focus on algorithms may actually be harmful to children, since it distorts their beliefs about the nature of mathematics—that mathematics is primarily memorizing rather than reasoning (Burns, 1994; Kamii & Lewis, 1993; Mandell, 1985)." (Huinker, 1998, p. 170)

The third stage involved in the problem solving process is carrying out the plan. This stage should not require much effort provided that the work has been done properly in the first two stages. Students need to carry out the plan that has been laid out in the second stage, watching for any errors or shortfalls that may arise (Polya, 1945).

The final stage in problem solving is looking back. In this stage, it is important to check the solutions for reasonableness. Did the solution they came up with make sense? If not, they should check why. It is also important to have the students check to see if there would be another, perhaps more efficient, way to come up with the same solution (Polya, 1945).

*Examples of Problem Solving Strategies*

A study on learning new problem-solving strategies describes two methods of teaching students to solve equations. In an experiment of testing the two methods, 5th grade students were divided into four groups: no instruction (the control group), received add-subtract instruction, equalize instruction, and both add-subtract and equalize instruction. The group of students in the add-subtract group used an algebraic process to

solve the equation. For example; if a student was to solve for x in the equation x + 5 = 10 they were taught to take 10 minus 5 in order to solve for x. Another example would be to solve x-6=8 the students were told to take 8 plus 6 to solve for x. The students were not taught what everything meant but were taught how to find the answer (Alibali, Phillips, & Fischer, 2009).

The group that received equalize instruction used a process that included understanding what the equal sign meant in the equation. For example; to solve the equation x-5 = 10 they were taught that if you have x minus 5 on one side of the equal sign and you want to get x alone you need to add 5 to that side. In order to keep both sides equal then 5 must be added to the other side of the equals sign. The study found that understanding the concept of the equal sign allowed for better retention and better application when moving on to more advanced topics. This study shows the importance of Polya's first stage (understand the problem) in problem solving (Alibali, Phillips, & Fischer, 2009).

Another technique in developing problem-solving is questioning. Students need to be asked questions that will allow them to develop proper problem solving skills. The instructor must be careful not to ask the question in the wrong way so they do not give away too much information. A student must be asked a question in a manner that allows them to think critically and do some of the work involved. If too much information is given, the student will not get much out of the problem and their skills in problem solving will not be developed (Polya, 1945).

A study conducted on students in Taiwan used computer games to see if types of questioning made a difference in problem-solving skills of students. The study also

wanted to find out if prior knowledge makes a difference in developing problem solving skills. The results showed that using more specific types of questioning helped develop their problem solving skills. They also found that students with a high level of prior knowledge scored higher in the area of problem solving than students with low prior knowledge (Lee & Chen, 2009).

*Can this Reduce Enrollment in Remedial Mathematics Courses?*

Enhanced Anchored Instruction (EAI) is a method of mathematics instruction designed specifically for students with learning disabilities that has its basis in teaching students problem solving in Bottge's studies. According to the Teaching Enhanced Anchored Mathematics Team 2, EAI "situates problems in engaging contexts, strengthens foundation skills, taps background knowledge, promotes learning transfer, encourages problem-solving discussion, uses multimedia and hands on instructional tools, and encourages interdisciplinary teaching." (Bottge, n.d.).

In one study, Bottge compared students in Remedial Mathematics to students in a high achieving Pre-Algebra course. Prior to the study, the scores of the students in the Remedial Mathematics class were lower than the students in the Pre-Algebra course. After using EAI instruction, their scores were statistically the same on a problem solving test. The computation scores for the Remedial Mathematics students remained lower than the students in the Pre-Algebra class, but when looking at classroom observations, student comments and teacher interviews, the students in the Remedial Mathematics course had a deeper understanding of the nature of the problems (Bottge, Heinrichs, Chan, & Serlin, 2001).

Dr. Bottge also used EAI to conduct research on various Learning Disabled (LD) and non Learning Disabled students. The goals of this study were to see how teaching using a problem solving based approach is beneficial for both LD and non LD students. The study looked at improvement of computation scores and problem solving abilities for both sets of students (Bottge, Heinrichs, Chan, & Serlin, 2001). At the beginning of the study, there was a gap in mathematical ability between the groups of students. After going through the study, both groups improved; the LD students still tested below the non-LD students and also the ability gap between them remained the same (Bottge, Rueda, Serlin, Hung, & Kwon, 2007).

Another Bottge study, compared students learning through video based EAI instruction to students learning from a text based instruction (TBI). The study included both LD and non-LD students. The study found that students learned well in both TBI and in EAI instruction. The TBI students tested better on word problems while the EAI students tested better on video problem solving. A study was conducted a few months later to check for retention and the study found that the EAI students had a much better retention rate. Additionally, in an observation of two of the LD students, the study found that these students who did not previously enjoy studying mathematics had developed a desire to learn and use mathematics skills (Bottge, Heinrichs, Mehta, Rueda, Hung, & Danneker, 2004).

Bottge stated “Curriculum and teaching methods too often do not meet the learning needs of many students” (Bottge, 2001, p. 68) in response to a superintendent when the superintendent was wondering why so many students are getting classified for special education. It can become difficult finding the difference between students with

learning disabilities and low-achieving students. Problem solving was a suggested method of instruction, using a video to get students engaged. The study found that, compared to students who received traditional instruction on the same topic, the problem solving students scored significantly better. Student behavior improved significantly; students were proud of the work they accomplished and discovered (Bottge, 2001).

*Will Student Attitudes Towards Mathematics Improve?*

A study on promoting critical-thinking dispositions was conducted to see if student attitudes towards mathematics improved when using a problem-solving based instruction. The study looked at various aspects that were found to be important for student improvement in attitude. One aspect received in problem solving based instruction was direct experience. In one example of problem solving based instruction, a story about eagles was used. Students became engaged in the story and had to solve problems similar to the ones in the story. They had a sensory experience through a video using sights and sounds. They had an emotional tie by discussing how to best save an eagle. The students had to define the problem to be solved and make choices as to how to solve the problem that supported their freely chosen method of saving the eagle. The result of the study include that the development of attitude strength takes time, students need to have a stake in their learning, and multiple perspectives on problem solving have to be built into the design (Leader & Middleton, 2004).

Another study looked at the effects of using SQUARE ONE TV on elementary students. SQUARE ONE TV is a television series that uses acting to teach students problem solving based mathematical skills. The study found that students who watched the television program showed a significantly greater ability in using problem solving

skills compared to the non-viewing students. It was observed that students who watched the television program talked more about enjoying mathematics class. They looked at mathematics as an "active process of thinking and reflection, not as a rote or formulaic procedure" (Fisch, et al., 1991, p. 13).

A study conducted in Taiwan looked at the effects of using personalized computer based instruction. This study was conducted because of problems with students' willingness to solve word problems and also to improve their attitudes towards mathematics. The study included two groups: one was taught with a personalized computer based program and one with a non-personalized computer based instruction. The students with personalized instruction were posed questions as they learned. The study was founded on the basis that motivation is an important factor in solving word problems. The study found that students in the personalized group not only had better attitudes towards mathematics but also tested statistically higher on the post-test than the non-personalized group (Chen & Liu, 2009).

*Will Test Scores Improve?*

Problem solving is one of the NCTM Process Standards (National Council of Teachers of Mathematics [MCTM], 2000). The NCTM wants students to build mathematical knowledge through problem solving, solve problems in mathematics, apply and adapt a variety of strategies to solve problems and reflect on the process of mathematical problem solving (MCTM, 2000). Written in the content standards it says:

> Solving problems is not only a goal of learning mathematics but also a major means of doing so. It is an integral part of mathematics, not an isolated piece of the mathematics program. Students require frequent opportunities to formulate,

> grapple with, and solve complex problems that involve a significant amount of effort. They are to be encouraged to reflect on their thinking during the problem-solving process so they can apply and adapt the strategies they develop to other problems and in other contexts. By solving mathematical problems, students acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that serve them well outside the mathematics classroom. (NCTM, 2000, p. 4)

A study was conducted to make a comparison on problem solving skills using two different schools. One school taught with a traditional curriculum and the other used an NCTM orientated curriculum called Core-Plus. The conductors of the study wanted to test the levels that students reached in the four stages of problems solving so they administered their own test orally to find out. The researchers took the two groups and gave mathematical problems that tested their problem solving skills. The students were videotaped and later observed for proper problem solving strategies used and scored on a rubric. Students in the traditional curriculum typically tried a straight forward approach to try and solve problems while the Core-Plus students tried many different strategies. With stage four of Polya's four stages of problem solving being the highest possible stage to reach, they found that the students in the NCTM orientated curriculum group would consistently reach stage three while the students in the traditional curriculum would only reach stages one or two. Neither group of students entered stage four (Latterell, 2003).

Bar-Ilan University in Israel conducted a study on students using three different approaches. The first approach was a diagnosis errors approach (DIA). The second approach was by doing improvement via self-questioning (IMP). The third approach was

conducted using both the DIA approach and the IMP approach. All three of these approaches were compared to a control (CONT) group that received no metacognitive instruction. The study found that the student in the combined approach was the most effective overall. The IMP approach outperformed the DIA students in both problem solving skills and metacognitive strategy use. The DIA students were better in reducing conceptual errors (Kramarski & Zoldan, 2008).

*Can the State Standards be Met Using Problem-Solving Instruction?*

Problem solving is a requirement of the MCTM and the NCTM. Below, there is a list of what is recommended concerning problem solving in the NCTM Process Standards, what is listed in the NCTM content standards, and what is listed as the goals of the Minnesota Council of Teachers of Mathematics.

*NCTM Process Standards* (NCTM, 2000):

"Instructional programs from prekindergarten through grade 12 should enable all students to-

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

*NCTM Content Standards* (NCTM, 2000):

"Problem solving: Solving problems is not only a goal of learning mathematics but also a major means of doing so. It is an integral part of mathematics, not an isolated piece of the mathematics program. Students require frequent opportunities to formulate, grapple with, and solve complex problems that involve a significant amount of effort.

They are to be encouraged to reflect on their thinking during the problem-solving process so they can apply and adapt the strategies they develop to other problems in other context. By solving mathematical problems, students acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that serve them well outside the mathematics classroom (NCTM, 2000).

*From MCTM* (About MCTM, 2010)**:**

The goals of the Minnesota Council of Teachers of Mathematics are as follows:

- To develop an active interest in the science of mathematics.
- To help provide opportunities for the exchange of ideas and materials regarding instruction in mathematics.
- To further the study of problems relating to the teaching of mathematics at the elementary, secondary, and college levels.
- To work for the improvement of mathematics instruction at the elementary, secondary, and college levels in Minnesota.
- To work for the improvement of employment and service of members of the Council and members of the profession in general.

*Other Findings*

One research article on at-risk students examined preventing them from dropping out of school. At-risk students are students that are on free and reduced lunch, have ill family members, live in poor housing, crime stricken areas, and may have no telephone or no family vehicle. One of the areas covered in the research was problem solving.

For these students, problem solving skills were essential for them to improve their situation. They could learn to avoid problem situations, and these skills allowed them to understand how they could graduate from high school. The article went as far to say:

> Students will use problem-solving skills their entire lifetime. While some students may learn them on their own, many do not. They must be taught the skills. Problem-solving skills require practice. As students practice, they get better at solving problems and at solving more complex problems. (Thurlow, Christenson, Sinclair, Evelo, & Thornton, 1995, p. 15)

They found the number of dropouts decreased, less students failed mathematics classes, more students earned credits, less absenteeism occurred, behavior ratings improved, attendance improved, and student satisfaction with school rose.

# Chapter 3: Interpretation

The research reviewed covers many aspects in regards to problem solving. These aspects can be very useful in the mathematics classroom if implemented properly. Some of the details of problem solving the author found have put things into perspective for him. The author has gathered information that will he will be able to share with other mathematics instructors as well as implement in his own classroom.

## *My Observation of Importance of Polya's Four Stages*

Going thoroughly through each of Polya's Four Stages of Problem Solving is extremely important. If students do not thoroughly understand in each of the stages, they will not be able to efficiently or effectively solve problems. This will not only affect their ability to do and understand mathematics, but they will also miss components of a skill that can be used across all curriculums. If, however, they learn how to thoroughly follow each stage, learners of all capabilities will have the potential to become excellent problem solvers, in mathematics and in life.

The first stage of problem solving is critical because studies like the ones conducted by Bottge have found that students need to be engaged, and care about what they are learning, in order to learn mathematics successfully. If this first stage is missed, if students do not care about what they are learning, they may attempt to go through the other three stages. However, not connecting that emotion as well as not understanding what needs to be solved, students will be able to only go through the motions, at best, with the other three stages. This will only enforce the idea that mathematics is about memorization and following certain algorithms. If they eventually make their way to the final stage (after skipping the first one), they are not going to be able to ask if their

answer makes sense, because they did not understand what they were looking for in the beginning.

After reaching an understanding of what they are trying to find in stage one, the second stage requires students to figure out which strategies to use to solve the problem at hand. Mathematically, the second stage is most important. If students know numerous formulas but never develop an understanding of when to use them, the formulas become useless. If students are unable to develop a plan, the rest of the stages are impossible to go through. When mathematical problems are presented on a standardized test, or when students endure problems in life, being unable to develop a plan will cause students to be searching for an answer without direction. If students are able to effectively develop a plan, the rest of the problem solving stages are likely to be successful.

Since Polya's third stage is the implementation stage, students would get nothing done without this stage. Typically, students can learn how to use this stage to fool people that they know all the other stages. The students could be given the algorithms without going through the first two stages but can learn the formulas and mathematical methods well enough to pass standardized tests without really knowing what they are doing. A former student of the author admitted to continuously helping another student through Calculus. That student had received a five on their AP Calculus test, but they had earned that by merely memorizing formulas and methods. Even though this stage can be faked by showing the ability to follow algorithms, there needs to be more care taken so students can demonstrate that they can do all the other stages. Students also need to understand that developing strategies for solving problems can take a while. Developing persistence in problem solving is an important skill for students to achieve.

In the fourth and final stage, students are called to reflect and also to think about alternate solution paths. Students should be questioned in this stage more often. In this stage, students are able to explain which methods they used, the uses of what they discovered, and the processes they went through. When teachers skip this stage, the depth of what is taught is limited. Students need to be able explain their solutions and processes to others so there is solid evidence that they truly understand what they discovered. If a teacher only requires students to go through the first three stages without going to the fourth stage, students can falsely show their understanding by mimicking a bunch of routine algorithms.

From the discussion above, it is easy to see the importance of each stage. If any of the four stages are omitted, students will not get a complete comprehension of what they are to accomplish. The four stages are essential for students to develop a thorough understanding and passion for mathematics.

*How Problem Solving Based Instruction Helps Students*

Problem solving based instruction helps students develop a deeper understanding than only teaching algorithms. Students will not only be able to understand what they are doing much better but they can develop a passion for mathematics that will let them know it is more than just a "bag of tricks."

Studies such as those conducted by Bottge, Heinrichs, Mehta, Rueda, Hung, and Danneker (2004), Fische et al. (1991), and Leader and Middleton (2004) have showed that if mathematics becomes more meaningful to students, they will enjoy what they are learning. As a result of their enjoyment, they will want to learn more, their tests scores will be higher, and they will start applying mathematics skills to their everyday life. If we

teach students to understand the concepts behind the mathematics they are doing rather than merely learning rote memorization techniques, students will be able to transfer what they have learned to the next lesson. The focus of each lesson they move on to will be learning mathematics, not learning to memorize the next group of formulas.

Video-based instruction and SQUARE ONE TV are two examples of ways today's technology is used to help students implement problem solving strategies in real-life scenarios. Having the real-life scenarios along with technology provides students with background knowledge of a problem and sparks student interest. If students care about the problem and are engaged in what they are doing, their problem solving potential will be maximized.

Using what Dr. Bottge found, we can trust that problem solving helps students at all levels. Studies have shown that traditional ways of teaching math only help some students learn and only sometimes help those students learn at an in-depth level. Since problem solving helps at all levels and develops a deeper level of engagement and understanding, we need to begin moving our curriculums in the direction of problem solving.

Proper questioning techniques have also been found to enhance students' understanding of mathematics. When questioned properly, students will develop the skill of thinking critically and as he mentioned in chapter two, students had a deeper understanding using a computer game when prompted by effective questions along the way. Perhaps teachers could learn how to offer questions to these students, and not allow them to depend on the teacher for answers. This would not only allow for critical thinking skills to develop. Students would gain more confidence in problem solving if they were

able to solve difficult problems through questioning, thereby allowing the student to reach the solution, rather than the teacher solving a problem for them.

In order to enrich their lives, students need to be taught how to problem solve. In the research he has found problem solving has many positive effects on student lives. If a teacher wants to enhance student education, problem solving skills is a key.

*Problem Solving in Mathematics Curriculum*

Every district wants their students to score higher on their state tests. Research such as conducted by Kramarski and Zoldan (2008) has shown that teaching using problem solving based instruction can do this. If more students are taught the way they need to learn and are more passionate about learning, test scores will improve.

Mathematics teachers need to raise the bar for education of our youth. Problem solving is listed as a goal in the NCTM content standards, the NCTM process standards, and as MCTM goals, which the author discussed in chapter two. Therefore, if teachers are following standards, problem solving should already be incorporated into their curriculum. The author believes that some teachers may not include problem solving, if any, in their classroom. This makes it even more of a challenge to encourage problem solving in the higher grades when it is not being included at all the previous grade levels. Problem solving needs to be included in the curriculum starting with kindergarten. Ideally, most mathematics lessons would include some aspect of problem solving – either teaching a lesson through the process of problem solving or teaching on the topic of problem solving itself. Since some classrooms may not even involve a portion of problem solving, Minnesota may seem a long way from including problem solving in every mathematics lesson.

To help Minnesota move in the right direction regarding problem solving, teachers need to be educated on the process of problem solving and the benefits thereof. If teachers become aware of the positive effects of problem solving instruction and if more problem solving resources were available, teachers would be more motivated to include problem solving in their every day instruction. Parents also need to be educated about problem solving, so that they can support the school as mathematics curriculum moves in the problem solving direction.

If students are taught to problem solve, this skill will be useful in other courses. What science teacher does not want their students to be able to problem solve? The new Minnesota science standards are requiring engineering coursework to be applied to the curriculum. Teaching problem solving applies directly to the career of engineering. Studies like the one conducted by Thurlow, Christenson, Sinclair, Evelo, and Thornton (1995) have found that children who learn to problem solve can have better personal lives.

# Chapter 4: Conclusion

In this section, the author has included some details about his classroom, a description on how he has used problem solving, and his observations on Polya's four stages of problem solving, how problem solving based instruction helps students, and how he will promote problem solving based instruction.

## *Details about the Author's Classroom*

The author teaches in a small school in northwestern Minnesota. Approximately 500 students, K-12, are housed in our building. His class size averages around fifteen students per class. The majority of the student body is Caucasian with a small percentage of American Indians and Asian Americans. They are a Title one school which includes having a large portion of their student body on free and reduced lunch.

The author teaches mainly sophomores, juniors, and seniors. The classes he teaches are Geometry, Algebra, College Algebra, Pre-Calculus, Calculus, Introduction to Mathematical Sciences, Digital Electronics, Intro to Engineering Design, Principles of Engineering, and Engineering Design and Development. The author has been a mathematics instructor for ten years.

## *Using Problem Solving Based Instruction in My Classroom*

Throughout the author's teaching experience, he has always used some form of problem solving in his classroom. It has come up in mathematical lessons, and he has consistently tried to get students to think about what they are doing. However, doing problem solving research has made the author aware that he has not always included all of the stages nor understood the necessity of learning to problem solve thoroughly and

effectively. The author reflected on what he has done with his students and recognized that he has taught them just pieces of how to problem solve.

The author plans to make a more formal approach to teaching problem solving. When he has designed lessons or activities in his classroom, he would find ways to incorporate problem solving in each lesson. Though the author has designed lessons to engage students in what they are learning, he now knows the importance of the four stages of problem solving, which he will discuss in the next section. Students need to present and report findings from their discoveries in mathematics.

*How the Author will Promote Problem Solving Based Instruction*

When mathematics is taught in school, a lot of teachers may not approach mathematics from a problem solving perspective. One of the big reasons teachers, the author included, do not take this approach is time. Teachers are always wondering, "Will I have enough time to cover everything? There is too much material to cover in so little time." Teachers feel they have to cram it all in. A lot of the curriculum districts teach is examples and a lot of practice problems without a practical approach to mathematics, which does not go through all the steps of problem solving. The author has had parents get upset when their children are not taught using traditional textbooks. Sometimes teachers fall back on teaching students a bunch of algorithms because it can be quicker and they want students to score well on tests. In some cases, teachers may have been used to learning mathematics by practicing problems. It would therefore be very difficult for them to use the problem solving method of instruction if they have a low understanding of the material themselves.

Another problem the author has found is changing students' minds. Not all students come to his class with a love for mathematics and knowing how to problem solve. He has noticed that when his students get into tenth grade and have not been taught how to use problem solving to learn mathematics, it is difficult to have them learn mathematics any other way than by algorithms. It is easy and what they are used to, so they just want to get it over with, memorize the formulas needed, get their grades, and be done. Students who have always thought they were good at mathematics may realize that they were only good at algorithms without ever understanding the content. Maybe the students that are truly good at mathematics are those who learned how to problem solve earlier in their lives.

*Findings*

Before conducting this research, problem solving to the author was nothing more than simply finding answers to mathematical problems in a mathematics class and teaching students how to do things mathematically. However, he found that problem solving is not only finding the answers but a process that students need to go through. As a result of this paper, he desires to change the way he teaches mathematics, he wants to start encouraging elementary teachers to start incorporating problem solving, and he hopes to explain to others the effects it can have on children in poverty.

Before conducting the research, the author did not know the process that students have to go through in learning to problem solve. Maybe it seems that mathematics lends itself to that process naturally but the author discovered that it is a process that needs to be taught. As a teacher, if he is not able to engage his students, it will be very difficult to get them to understand what he is teaching them. The author needs to develop richer

lessons that spark student interest in order for them to desire to find a solution. He does this currently in some lessons, but knows that he needs to do this more in his classroom. The author needs to make sure he does not give students problems that are harder than they can handle so they do not lose confidence and want to keep trying to solve the problem. He discovered that he is guilty of creating problems more difficult than their capabilities some of the time. The author always wants to challenge all of his students, and he needs to keep doing that, but he also needs to make sure it is not beyond their ability. The author also realized that he follows the final stage of the process sometimes, but he now realizes the importance of this stage. Students need this stage to solidify what they have learned, be able to explain and display their findings, and feel proud of what they have discovered.

The author has become an adamant believer in the process of problem solving and wants to make sure he incorporates it in developing lessons. He has recently talked with his school's fifth grade teacher about the problem solving process, and he would like to work with other elementary teachers so the problem solving process starts being taught at a very young age. If the elementary teachers are willing, the author plans on helping them develop lessons that incorporate problem solving in their mathematics lessons. The author truly believes this process needs to start being incorporated at a young age and used throughout their learning experience.

After reading the research article on problem solving and its effects on students in poverty, the author knew the problem solving process needed to be enhanced in his district. This resonates with the author as he knows his district is always looking for ways to keep students in school. If problem solving skills are taught starting at a young

age and continue to be taught throughout school that could make a tremendous impact on these students' lives. The author's school has over fifty percent of students on free and reduced lunch. Effectively teaching problem solving skills to his students can make a vast difference in their lives. Students of poverty may not always get taught important life skills, teaching problem solving becomes even more important to these students.

In conclusion, the author needs to continue to make improvements in the way he teaches and in the way mathematics is taught in our district. He now has a process to follow and knows how important it is to use this method. He knows the impact it can have on student learning and will be able to develop criteria on developing lessons that encourage the process of problem solving.

*Call for More Research*

The author would like to examine more research on how to effectively teach problem solving to all students. He would like to analyze some longitudinal studies among students taught problem solving from kindergarten through twelfth grade. The author would like to see research discussing if there is a mathematical ability gap between students in poverty and those not in poverty. If research shows an achievement gap, the author would like to examine if problem-solving skills can help students in poverty reach the mathematical skills of those not in poverty. The author would like to see research on how to teach other teachers how to effectively teach problem solving.

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Bottge, B. A., Heinrichs, M., Chan, S.Y., & Serlin, R. (2001). Anchoring adolescents' understanding of math concepts in rich problem-solving environments. *Remedial and Special Education, 22*(5), 299-314.

Bottge, B. A., Heinrichs, M., Mehta, Z. D., Rueda, E., Hung, Y.H., & Danneker, J. (2004). Teaching mathematical problem solving to middle school students in math, technology education, and special education classrooms. *Research in Middle Level Education Online, 27* (1), 1-17.

Bottge, B. A., Rueda, E., Serlin, R. C., Hung, Y.-H., & Kwon, J. M. (2007). Shrinking the achievment differences with anchored math problems: Challenges and possibilities. *The Journal of Special Education, 41*(1), 31-49.

Bottge, B. (2001, March). Using intriguing problems to improve math skills. *Educational Leadership*, 68-72.

Bottge, B. (n.d.). *What is enhanced anchored instruction (EAI)*. Retrieved March 13, 2010, from http://edsrc.uky.edu/TEAM2/what_is.html

Cawley, J., Parmar, R., Maccini, P., Gagnon, J., Thurlow, M., Montague, M., et al. (2002). *Strengthening the third "R": Helping students with disabilities achieve in mathematics.* Arlington, VA: The ERIC Clearinghouse on Disabilities and Gifted Education.

Chen, C. J., & Liu, P. L. (2007). Personalized computer-assisted mathematics problem: Solving program and its impact on taiwanese students. *Journal of Computers in Mathematics and Science Teaching*, 105-121.

Engineers Guide USA. (2009). *Engineer Job Outlook*. Retrieved 3 27, 2010, from Engineer Guide USA: http://www.engineersguideusa.com/Careers/engineer_career_outlook.htm

Fisch, S. M., Hall, E. R., Esty, E. T., Debold, E., Miller, B. A., Bennett, D. T., et al. (1991). *Children's problem-solving behavior and their attitudes toward mathematics a study of the effects of square one tv.* New York: Children’s Television Workshop.

Gonzales, P., Williams, T., Jocelyn, L., Roey, S., Kastberg, D., & Brenwald, S. (2009). *Highlights from TIMSS 2007: Mathematics and science achievment of U.S. fourth- and eigth-grade students in an internatinal context.* Washington, D.C.: National Center for Educational Statistics.

Huinker, D. (1998). Letting fraction algorithms emerge through problem solving. *Yearbook (National Council of Teachers of Mathematics),* 170-182.

Kramarski, B., & Zoldan, S. (2008). Using errors as springboards for enhancing mathematical reasoning with three metacognitive approaches. *The Journal of Educational Research,* 137-151.

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Leader, L. F., & Middleton, J. A. (2004). Promoting critical-thinking dispositions by using problem solving in middle school mathematics. *Research in Middle Level Education Online, 28*(1), 1-13.

Lee, C. Y., & Chen, M. P. (2009). A computer game as a context for non-routine mathematical problem solving: The effects of type of question prompt and level of prior knowledge. *Computers & Education,* 530-542.

McGehee, J. J. (2001). Developing interdisciplinary units: A strategy based on problem solving. *School Science and Mathematics, 101,* 380-389.

Minnesota Council of Teachers of Mathematics. (2010). *About MCTM.* Retrieved March 11, 2010, from http://www. mctm.org/about.php

National Council of Teachers of mathematics. (2000). *Process Standards*. Retrieved March 12, 2010, from NCTM: http://www.nctm.org/standards/content. aspx?id=322

Neurath, R. A., & Stephens, L. J. (2006). The effect of using microsoft excel in a high school algebra class. *International Journal of Mathematical Education in Science and Technology,* 721-756.

Niemi, D. (1996). Assessing conceptual understanding in mathematics: Representations, problem solutions, justifications, and explanations. *The Journal of Educational Research, 89,* 351-363.

Polya, G. (1945). *How to solve it.* Princeton, NJ: Princeton University Press.

Schoenfeld, A. H. (1987). Polya, problem solving, and education. *Mathematics Magazine, 60* (5), 283-291.

Thurlow, M., Christenson, S., Sinclair, M., Evelo, D., & Thornton, H. (1995). *Staying in school: Strategies for middle school students with learning & emotional disabilities.* Minneapolis, MN: Institute of Community Integration.

# Annotated Bibliography

Alibali, M. W., Phillips, K. M., & Fischer, A. D. (2009). Learning new problem-solving strategies leads to changes in problem representation. *Cognitive Development,* 89-101.

This study teaches students different strategies to solve mathematical problems. The researched the effects of teaching no strategies, one strategy, and two strategies. They also wanted to see if one strategy is better than another. This will be helpful in giving different strategies and their effects.

Bottge, B. A., Heinrichs, M., Chan, S.-Y., & Serlin, R. (2001). Anchoring adolescents' understanding of math concepts in rich problem-solving environments. *Remedial and Special Education, 22*(5), 299-314.

In this article the authors check the outcomes of problem solving skills and computation skills using a problem solving approach to teach mathematics. Computation skills do not really improve but the understanding of the problem improves. This will be helpful to emphasize the importance stage 1 of problem solving.

Bottge, B. A., Heinrichs, M., Mehta, Z. D., Rueda, E., Hung, Y.-H., & Danneker, J. (2004). Teaching mathematical problem solving to middle school students in math, technology education, and special education classrooms. *Research in Middle Level Education Online, 27* (1), 1-17.

This article explains how you can spark interest in students to teach mathematics. It will be useful in the area of improving students' math skills.

Bottge, B. A., Rueda, E., Serlin, R. C., Hung, Y.-H., & Kwon, J. M. (2007). Shrinking the achievment differences with anchored math problems: Challenges and possibilities. *The Journal of Special Education, 41*(1), 31-49.

This study was conducted on high achieving students in a pre-algebra course using problem solving based instruction and compared to learning disabled students using a problem solving based instruction. The study found out that both sets of students improved equally. This will be very useful to show that using problem solving base instruction could prevent learning disables students from getting further and further behind.

Bottge, B. (2001, March). Using intriguing problems to improve math Skills. *Educational Leadership*, 68-72.

Bottge begins this article by explaining that too many students are classified as special education students. This study uses problems solving to teach students to solve math problems using a video based launch to get students engaged. This study will be very

useful in showing how in isn't necessarily that the students have math disabilities but that they are taught incorrectly.

Bottge, B. (n.d.). *What is enhanced anchored instruction (EAI)*. Retrieved March 13, 2010, from http://edsrc.uky.edu/TEAM2/what_is.html

This website explains EAI instruction. This will be useful to help explain how problem solving instruction is used.

Cawley, J., Parmar, R., Maccini, P., Gagnon, J., Thurlow, M., Montague, M., et al. (2002). *Strengthening the third "R": Helping students with disabilities achieve in mathematics.* Arlington, VA: The ERIC Clearinghouse on Disabilities and Gifted Education.

This is a publication on the need of following the national math standards. How students need to learn mathematics by use of problem solving to enhance their education and compete with other countries.

Chen, C. J., & Liu, P. L. (2007). Personalized computer-assisted mathematics problem: Solving program and its impact on taiwanese students. *Journal of Computers in Mathematics and Science Teaching*, 105-121.

This study uses computer programs to teach students how to problem solving. The study found out using proper questioning techniques will help students develop problem solving skills. I will use this article to emphasize the importance of proper questioning techniques.

Engineers Guide USA. (2009). *Engineer Job Outlook*. Retrieved 3 27, 2010, from Engineer Guide USA: http://www.engineersguideusa.com/Careers/engineer_career_outlook.htm

This is a source on careers for people that are problem solvers. The site covers the need of future Engineers divided up by different fields. Also covered on the site is the types of skills needed to work in each field of study.

Fisch, S. M., Hall, E. R., Esty, E. T., Debold, E., Miller, B. A., Bennett, D. T., et al. (1991). *Children's problem-solving behavior and their attitudes toward mathematics a study of the effects of square one tv.* New York: Children's Television Workshop.

This study was to study the effects of a problems solving based television show on students math skills. The found out students that watched the television show talked more positively about mathematics. I will use this article to show positive effects of problems solving on student attitudes towards mathematics.

Gonzales, P., Williams, T., Jocelyn, L., Roey, S., Kastberg, D., & Brenwald, S. (2009). *Highlights from TIMSS 2007: Mathematics and science achievment of U.S. fourth- and eigth-grade students in an internatinal context.* Washington, D.C.: National Center for Educational Statistics.

This is an article summarizes the testing on mathematics scores from different countries around the world.

Huinker, D. (1998). Letting fraction algorithms emerge through problem solving. *Yearbook (National Council of Teachers of Mathematics),* 170-182.

This is a study that uses real life situations to teach students fractions. It has some positive ways to teach fractions to deepen the students understanding of fractions. This study will be useful to explain another method of using the stages of problem solving to teach mathematics in the classroom emphasizing on the first stage.

Kramarski, B., & Zoldan, S. (2008). Using errors as springboards for enhancing mathematical reasoning with three metacognitive approaches. *The Journal of Educational Research,* 137-151.

This study uses two approaches to teach problem solving to students in Israel. The found out that Using problem solving methods in their classrooms improved education versus no problem solving. This article will be useful to show importance of using errors to teach mathematics.

Latterell, C. M. (2003). *NCTM-orientated versus traditional problem-solving skills.* Baltimore, MD: EDRS.

This research paper found that students in NCTM orientated curriculum achieved further stages in problem solving than students in traditional curriculum. This study will define the importance of teaching NCTM curriculum to enhance problem skills.

Leader, L. F., & Middleton, J. A. (2004). Promoting critical-thinking dispositions by using problem solving in middle school mathematics. *Research in Middle Level Education Online, 28*(1), 1-13.

This article uses a activity based method to teach problem solving in a classroom. One of the finding in this article is how students' attitudes towards mathematics are affected. This article will be very useful to show how attitudes can be changed when problem solving is taught.

Lee, C. Y., & Chen, M. P. (2009). A computer game as a context for non-routine mathematical problem solving: The effects of type of question prompt and level of prior knowledge. *Computers & Education,* 530-542.

This is a study that used computer based questioning techniques to teach problem solving. The study found that using proper questioning techniques to teach mathematics through problems solving was beneficial. This study will be useful to explain techniques of enhancing problem solving skills.

McGehee, J. J. (2001). Developing interdisciplinary units: A strategy based on problem solving. *School Science and Mathematics, 101*, 380-389.

This article describes problem solving and the interdisciplinary uses of problem solving. It will help further the explanation of problem solving and extra uses across other curriculums.

Minnesota Council of Teachers of Mathematics. (2010). *About MCTM.* Retrieved March 11, 2010, from http://www. mctm.org/about.php

This web page describes the philosophies of the Minnesota Council of teachers of Mathematics.

National Council of Teachers of mathematics. (2000). *Process Standards*. Retrieved March 12, 2010, from NCTM: http://www.nctm.org/standards/content. aspx?id=322

This site explains the current national mathematics standards.

Neurath, R. A., & Stephens, L. J. (2006). The effect of using microsoft excel in a high school algebra class. *International Journal of Mathematical Education in Science and Technology*, 721-756.

This study uses computer software to teach high school algebra. The study found that test scores do not statistically improve but students' attitude towards mathematics does. This finding will be useful in defining tools to use to improve students attitudes.

Niemi, D. (1996). Assessing conceptual understanding in mathematics: Representations, problem solutions, justifications, and explanations. *The Journal of Educational Research, 89*, 351-363.

This study took a group of inner city students and tested their ability in problem solving. There is some useful information on the necessity of teaching problem solving compared to rote memorization.

Polya, G. (1945). *How to solve it.* Princeton, NJ: Princeton University Press.

This is the text that describes problem solving based instruction. Polya discusses his findings on proper techniques to use in teaching problem solving. This text will be very useful as a basis of understanding problem solving based instruction and how it should be used.

Schoenfeld, A. H. (1987). Polya, problem solving, and education. *Mathematics Magazine, 60* (5), 283-291.

This is an article on the importance of teaching problem solving and the methods discussed by Polya.

Thurlow, M., Christenson, S., Sinclair, M., Evelo, D., & Thornton, H. (1995). *Staying in school: Strategies for middle school students with learning & emotional disabilities.* Minneapolis, MN: Institute of Community Integration.

This article discusses the positive effects teaching problem solving can have on their personal lives and their future. This article will help show other uses for teaching problems solving