How Geometer's Sketchpad Improves Student Learning

by

William George Awe

ABSTRACT:

This study investigated whether dynamic software improves learning and reviews different types of dynamic software. The study then narrows its focus to one type of dynamic software, Geometer's Sketchpad and studies how Geometer's Sketchpad, in itself, improves student learning.

The researcher identified what dynamic software is, the importance of dynamic software and different types of dynamic software. The researcher also identifies how dynamic software is utilized in schools and researches how to improve the use of dynamic software in schools.

Dynamic software, and Geometer's Sketchpad, In particular, was found to improve learning across grade levels and with different levels of learners, while still allowing the user to probe deeper into questions and further critical thinking, since the users are allowed to learn at their own pace, so that all students can continue learning about a subject and test different hypotheses that they want to investigate.

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CHAPTER 1: INTRODUCTION

Setting

Teaching math has progressed throughout time as new devices and methods have been introduced. The challenge has been to identify which devices and methods are helpful and which devices and methods are just different. Change for change's sake is not educationally correct but a change for education's sake is correct. Teachers need to find the most effective way to teach, whether it is "old way" or "new way".

From the abacus (used in 500 B.C. by the Greeks) to the slide rule to the calculator, there have been questions of when/how to teach using new "mathematical inventions", that go on today with the basic calculator and the new graphing calculator. Part of the reason is that everything is being improved upon; consider how many different calculators have been "invented" that do various things. In addition, remember that from the time the slide rule was first "invented" by William Oughtred in 1632, it took until the 1650s for Victor Mayer Amedee Mannheim to make it into its present form. Another eight documented changes were made to it by 1940, the last being in 1936.

The 1970s and 1980s saw calculators and computers entering schools and being placed into school curriculum. Educators' challenges since then have been to incorporate such tools in their curricula so that we are producing positive change. Every new program or calculator is evaluated for its educational usefulness in the classroom for that specific teacher. These changes and new programs will continue to be made to help education, just as the slide rule, etc… was changed many times.

Each school has a goal of producing better students each year and teachers work very hard to raise achievement levels so that this goal can be accomplished. Dynamic software is important because it forces students to interact **with** it, which should produce a greater understanding of what is being worked on. One tool that can be used to accomplish this goal is dynamic software. The software that I have found to be worthwhile at a variety of grade levels and subjects, while still being able to teach and reach students in a way that can't be done as effectively in any other way, is Geometer's Sketchpad, which is published by Key Curriculum Press.

The National Council of Teachers of Mathematics (NCTM), in its publication, Curriculum and Evaluation Standards for School Mathematics, states, "The new technology not only has made calculations and graphing easier, it has changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate them." Later in the publication the NCTM says, "students should learn to use the computer as a tool for processing information and performing calculations to investigate and solve problems".

With the emergence of computer technology in most homes, mathematics educators feel computer software is an effective tool to increase learning. Due to lower costs, more homes now have computers and computer software. Because of this, parents are seeing how their children can learn some things better with technology and want the schools to readily adopt this attitude also. The NCTM has even made Technology one of its six Principles of Mathematics, in its publication Principles and Standards in Mathematics, saying "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning."

Statement of the Problem

This paper will look at dynamic software and focus on Geometer's Sketchpad. This paper will investigate whether GSP is a worthwhile teaching tool for raising achievement levels and improving learning. This paper will also investigate whether dynamic software, and Geometer's Sketchpad in particular, can be used at various levels of instruction.

This research paper will use current research to answer the following questions:

- Can Dynamic Software Improve Student Learning?
- Can Geometer's Sketchpad promote learning with different levels of learners?
- Does Geometer's Sketchpad help students probe deeper into questions and further critical thinking?
- Can Geometer's Sketchpad be used as an effective tool to help students learn more?

Definition of Terms

The research examined will focus on software used to teach mathematics in grades 7-12. In particular, it will look at Geometer's Sketchpad, sometimes referred to as GSP or as Sketchpad. The term classroom will mean a mathematics classroom. NCTM is the abbreviation used for the National Council of Teachers of Mathematics. Dynamic software is software that allows you to change factors and variables and see results immediately. OELE is the abbreviation for open-ended learning environments, which are where students are provided varying amounts of help and support on an individual basis to help them reach their learning goals. A Lenart Sphere is a sphere that is used as a teaching tool in studying spherical geometry and its properties.

CHAPTER 2: DYNAMIC SOFTWARE

What is Dynamic Software?

Finzer and Jackiw (1998) defined Dynamic manipulation environments as being characterized by three attributes:

First, the program allows direct manipulation. When the user "drags" a point on the screen, they perceive that they are moving the point, not just the mouse. You feel inclined to say something like "I'm moving this point which is changing the triangle's vertex", rather than saying "I'm moving the mouse which drags the small circle on the screen, which changes the coordinates of the vertex".

Second, the program implements continuous motion. Here there is no lag time between the movement of the mouse and the movement of the point. When I change the vertex of a triangle, the size length and area of the triangle are changed also at the same time.

Third, the program is an immersive environment. Items on the screen are realistic and behave like their counterparts in the real world, giving the user the feeling of involvement in the program. You are focused on accomplishing what you are seeking rather than driving the technology.

Technology's Role

Yelland (1999) explored the potential of computer technology in classroom environments. The role of technology in elementary schools was considered and Yelland's study suggests that computers will not be used effectively in educational contexts unless five conditions are met. These are that technology must be integrated into curricula in environments characterized by active learning, inquiry and problem-solving where higher order thinking skills are promoted. We need to use technology to present and represent ideas, and we need new definitions of play and what constitutes a manipulative. Finally, it was suggested that both staff and students need to develop critical media literacy skills in their use of the new information technologies.

Yelland (1999) talks about this, saying "In order for computers to have an impact on existing curricula and pedagogy, changes will need to be initiated with both teachers and policy makers participating in the process. Yelland goes on to talk about society and how they've handled the changes in technology. "Our society has changed dramatically in the way that we do business, communicate, and access information. Meanwhile, our schools seem to have been frozen in a time warp with change coming slowly and reluctantly, so much so that our experiences in them are often very different from the experiences of life outside. We have available to us computer hardware and software that can help learners to shape ideas in new and original ways. Unfortunately, we still seem ill prepared as a profession to take advantage of opportunities that are available via this technology. In order for computers to have an impact on existing curricula and pedagogy, changes will need to be initiated with both teachers and policy makers participating in the process. The results will be evident in more meaningful and engaging learning for children in our schools that incorporate the use of technology, in all its manifestations.

Gningue (2003) believed that "computing-technologies have the potential for wide-ranging and long-lasting impact in the mathematics classroom". Potential, however, does not educate by itself. In his conclusion, Gningue states, "In addition, almost all teachers in both the course and workshops expressed this lack of administrative support. Most schools did not have a sufficient number of graphing calculators for all teachers to use at will. In some instances, teachers were not even aware that graphing calculators were available at their school.

The Importance of Dynamic Software

Yelland (1999) shows the impact computers can have on learning when he says, "All of a sudden learning by doing became the rule rather than the exception. Since computer simulation of just about anything is now possible, one need not learn about a frog by dissecting it. Instead, children can be asked to design frogs, to build an animal with frog-like behavior, to modify the behavior, to simulate the muscles, to play with the frog."

Yelland (1999) goes on to say, "Parents are now demanding that schools provide their children with access to the technologies that are appropriate to their child's effective learning. Anyone interested in education should be demanding not just the placement of computers in classrooms, but also that the machines be used to their full potential. At the present time teachers need more information about ways of using computers creatively in their programs, yet competing elements from other areas of the curriculum mean that they do not often have the time or opportunity to find out how computers could be incorporated into the curriculum more effectively."

Gningue (2003) studied two professional development experiences for middle and high school mathematics teachers: one long-term, the other short-term. The training of the long-term group (n=12) took place over an entire semester, in a 15-week, 45-hour graduate course, at an urban institution in New York City, that accented the use of computing technologies, especially the "TI83 Plus" graphing calculator and the "Geometer's Sketchpad," to enhance the teaching of mathematics in secondary schools. The training of the short-term group (n=11) took place in a series of three workshops totaling 7 hours, with teachers from the institution's Professional Development School, using essentially the same types of technology tools.

Gningue concluded that "A long, sustained, and more coherent form of training would have provided more opportunities for active learning, and could have lead PDS teachers to report more favorably about their experience with technology. In contrast, because course participants were provided with more opportunities for active learning, they all reported a change in beliefs and attitudes toward the use of technology in the classroom, as evidenced by the comments."

The NCTM (2000) Standards Teaching Principle says "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well." In other words, make sure the technology adds to the learning.

Manouchehri (1998) in his conclusion remarks that the goal of mathematics instruction at the middle grades, as acknowledged by the National Council of Teachers of Mathematics in 1999, is one of improving students' mathematical thinking and their ability to communicate thoughts more clearly. Later, Manouchehri says that "to fulfill these goals, any innovative geometry curriculum for the middle grades should capitalize on building students' geometric intuition by using a variety of useful tools for the experimentation."

Edwards (Spring 2005) used a three-day series of geometry activities and conversation starters for secondary school students. Using Dynamic Geometry Software, this series of activities encouraged students to construct their own conjectures and theories about locus points and function plots in geometry. Edwards concludes from his study that he has provided a glimpse of ways in which dynamic geometry software may be used as a tool to promote mathematical conversations with students -- both in whole group and one-on-one settings. The tools may be used to encourage a view of school geometry as a creative, engaging discipline -- one in which investigation, discussion, and analysis are central to all classroom activities. Rather than promoting the all-too-popular notion of secondary school geometry as a laborious subject remembered primarily for the "memorization of formulas" and "writing of two column proofs," dynamic geometry software enables classroom teachers to foster a view of geometry as an academic discipline in which questions are as important as answers and in which explaining one's ideas rigorously is as important as writing two-column proofs. It is in this manner that dynamic geometry software may help to break the cycle of misconception that seems to continually plague popular views of school geometry.

Santos-Trigo (2004) found that the role of dynamic software becomes an important tool for students to guide the exploration of mathematical relationships. In some cases, the use of the software provides evidence about the existence of particular relationships. Later on, Santos-Trigo says that " In general, the process of analyzing parts of certain geometric configurations represents a challenge for students allowing them to observe and document the behavior of families of objects (segments, lines or points) within a dynamic representation. Students themselves get the opportunity to reconstruct or discover new theorems or relationships. A crucial aspect that emerged in students' problem solving instruction is that with the use of dynamic software they had the opportunity to engage in a way of thinking that goes beyond reaching a particular solution or response to a particular problem."

In Santos-Trigo's remarks at the end he says that "An important goal of mathematical instruction is to provide an environment for students in which they have an opportunity to demonstrate their own ideas on ways to deal with mathematical problems. These initial ideas need to be challenged and expanded and the use of technological tools seems to offer an important means to meet this goal. In particular, students can construct distinct types of representations with the help of technological tools that are then able to be studied in terms of answering or discussing questions or issues posed by the students themselves. In this context, students can reconstruct mathematical relationships previously studied and also have the opportunity to identify new set of relationships."

Types of Dynamic Software

Even though there are many types of dynamic software to chose from, I am limiting my selection to math software, and in particular, Fathom Dynamic Data Software (Fathom), TinkerPlots Dynamic Data Exploration (TinkerPlots), Cinderella, and Geometer's Sketchpad.

Fathom

Fathom, according to Grandgenett (2005), "is relatively intuitive" and "has a user friendly drag and drop approach to creating tables, graphs, and inferential tests, which makes it easy to create a statistical analysis and display the information on a printable page". Grandgenett (2005) highlights a major strength of Fathom by talking about the "ease with which data can be represented in a variety of ways: a click and a drop will create a table of cases versus attributes for a collection, and a further click and drag will create an empty graph". Grandgenett (2005) further says, "Fathom is excellent when it comes to hypothesis tests and estimations of population parameters." Finally, Grandgenett (2005), says that Fathom is a "very good piece of software which has the potential to enrich student learning." Schultz (2001) also states that this program "would be a great instructional tool. It is suitable for teachers of statistics from middle school through college level."

TinkerPlots

TinkerPlots is an inquiry-based software that according to Steinke (2005) "is in line with the recommendations of the National Council of Teachers of Mathematics Curriculum Standards". TinkerPlots, according to Matthews (2007), "is extremely easy to use and dynamic in nature. The major advantage to this interface is the speed with which one can compare a lot of data along different attributes". Steinke (2005) goes on to say that "TinkerPlots is a must for all grade 4-8 teachers who want to turn their students on to the power and beauty of data" and makes "data management exciting and meaningful for our students and teachers". Keller (2005) states another advantage when he states, "Another plus is TinkerPlots ability to import data from Microsoft's Excel." Keller (2005), in his conclusion states that TinkerPlots can be used beyond its target audience of grades 4-8 because "TinkerPlots is designed to make students think outside the book. Students predict, question, research, collaborate, analyze and problem solve, all by rearranging a series of dots, and watching patterns unfold."

Cinderella

Cinderella is one of the three main Java – technology based interactive geometry programs, along with Geometer's Sketchpad and Cabri Geometry. While all are supported behind publishing companies, according to Dye (Spring 2002), "each has a distinctive web presence based on Java applets." Cinderella is able to do some things that the other programs cannot do. For instance, according to Dye (Spring 2002), Cinderella "includes the ability to create a self-contained "exercise", an interactive page containing a geometrical problem, a space in which to solve it, and the options, or buttons, to achieve the solution. Dye recommends Cinderella over Cabri and GSP because it "provides access to free Web resources that introduce students in a meaningful and methodical way to the workings of the more powerful parent program."

Dye (Autumn 2002) states that Cinderella "provides a useful insight into Euclid's second proposition (to place at a given point (as an extremity) a straight line equal to a given straight line) by facilitating the creation of online interactive exercises." Dye (2002) also talks about the fact that Cinderella is able to deal with complex conjugates when two circles are drug apart.

Geometer's Sketchpad

Geometer's Sketchpad "allows students to construct precise figures and manipulate them interactively", according to Satterfield (2001), "helping them to develop mental models for thinking about geometric shapes and their properties". Satterfield also remarks that Geometer's Sketchpad is ideal for cooperative learning but also serves teachers well as a demonstration tool so that if the instructor has a limited amount of computers, students can still understand the visual aspects of Geometry. As for what age level, Geometer's Sketchpad works for, Satterfield says, "The package is intended for mathematics students in middle school, high school, and beyond, but can also be adapted by younger users."

Little (1999) used an activity that uses Geometer's Sketchpad at the Pre-Algebra level. Little's students are given six weeks to use Geometer's Sketchpad to create a Geometry Construction Manual using Geometer's Sketchpad. The instruction manual must contain the method of construction for each of the basic constructions, plus an enrichment item. Each page has a sequence of diagrams, in addition to written instructions. Grading is done based upon three components – Clarity of diagrams, Clarity of written instructions, and Overall organization and presentation. This is a good activity because it combines mathematics and technology in a way that forces the students to use technical writing to compose the instructions. Technical writing is an area all students need work in and this would be beneficial to any teacher.

Manouchehri et al. (1998) are big proponents of introducing dynamic software, such as Geometer's Sketchpad, into the curriculum at the Pre-Algebra level. They talk about a progression through Geometry at the Pre-Algebra level. They suggest starting with Free Exploration. The Free Exploration phase usually lasts about 2-3 days and is the time when the students get familiar with the basics of Geometer's Sketchpad. They are given handouts and work on their own, trying to see what will work and what won't work.

The next phase is semi-structured activities. The semi-structured activities are tasks which are planned to facilitate students' discovery of specific geometry relationships or theorems. The first tasks done here are working with pre-constructed models, which the students make changes to and find new ideas. The second tasks involve the students constructing the models with some instructions by the instructor. Again, the students try to find new ideas or prove old ones with their model. One task mentioned involves triangle sums. This is an easy one to do that can be done in a number of different ways. One way is to have the students prove the triangle sum with use of animation and a formula shown on the screen. Finally, we have Independent Explorations and Problem Solving. These are more open-ended in the instructions. An example is "*Given a triangle, locate the midpoints of the sides. Connect the adjacent midpoints and determine if a relationship exists between the area of the triangle defined by the midpoints and the original triangle.*" A problem like the one above can lead to other problems for the students to work on, such as asking the students to prove the answer via Sketchpad and via a typical Euclidean-type proof.

Cannon (1999) shows us a computer program (with handbook) that provides geometry activities for the middle school teacher. Cannon talks about a situation where she has only one computer in the classroom. However, Cannon is able to use the computer program and Geometer's Sketchpad to greatly enhance the learning of her students. According to Cannon, "many times the activities I chose from the collection proved to be jumping-off points for discussions involving higher levels of reasoning than heard in the usual seventh-grade classroom discussions on Geometry."

Grandgenett (Fall 2006) found a program called Algebra in Motion worthwhile. This program is used in conjunction with Geometer's Sketchpad. The program has a set of animations that, when used with Geometer's Sketchpad, are able to help teach Algebra a lot better.

Weeks (2000) reminds us that Geometer's Sketchpad is also an excellent way to graph functions and further explore the "What if…?" questions that occur when we change a variable or coefficient in a function. Weeks takes the reader step by step through graphing a function, introducing the Geometer's Sketchpad user into areas they might not have used, including Graph, Create Axes, Point on Object and Plot As (x,y). One could easily use a slider to further enhance this exploratory step into graphing with the use of the Geometer's Sketchpad program.

Cory et al. (2004) remind that almost anything can be done on Geometer's Sketchpad and can be shown in a manner that students learn about it rather than be told about it, which is what we should be doing anyway. They show us that Algebra could be further enhanced by having the Algebra students learn more about areas, polygons, and such using Geometer's Sketchpad to help with their visualization of the problems when they are doing word problems in their Algebra classes. This could also be done with a Geometry class.

Satterfield (2001), says that Geometer's Sketchpad helps students to develop mental models for thinking about geometric shapes and their properties. Cory et al. (2004) talks about how Geometer's Sketchpad can be used to teach Pre-Algebra subjects, such as areas, and shows how it can be implemented into the Algebra 1 curriculum, also.

Weeks (2000) uses Geometer's Sketchpad to teach graphs and graphing, while Manouchehri et al. (1998) shows how a progression of Geometer's Sketchpad could be. Also, Silver (1998) tackles the tough subject of whether Geometer's Sketchpad can be used to teach proofs?

A case study by Shaw, Durden and Baker (1998) agrees with Hannafin's (2004) conclusion from its case study. The case study by Shaw, Durden and Baker, examined the geometry learning of Amanda, a high school sophomore with cerebral palsy. To assist her learning, the three major accommodations of a self-taught curriculum, a computer program called Geometer's Sketchpad, and nontraditional assessment were made. The results indicated that Geometer's Sketchpad made the learning more accessible for Amanda. Working independently with the computer helped Amanda because it accommodated her need for a lot of time to study a figure and to process the visual information.

Forsythe (May 2007) did a study using pairs of students at a computer using Geometer's Sketchpad and found that the results of the test given after the first section of the work showed no significant difference between the target group and the control group. However, the results of the second test, given after the second section of the work (carried out in the Spring term), showed that the target group did do better than the control group, and this result was statistically significant.

"The value for the students of using Sketchpad was that, sitting in pairs at a computer, they interacted with the software and learned how to get the computer to produce the desired geometrical objects. This meant that they had to be systematic about the names and labels of geometrical objects and processes. If the computer did not produce what the students wanted, then they had to engage in problem solving to sort this out. They helped each other in their working pairs and also helped other pairs around the classroom. They engaged in discussing and explaining geometrical concepts. During lessons with the computer, the pupils' conversations tended to involve more mathematics and less inconsequential chatter. In this way, I feel that their understanding of the concepts was better than if they had learned geometry through paper-based tasks. However, mixing computer work with paper-based tasks is probably the best way to ensure the students can access the mathematics and enhance their understanding further."

Dynamic Software in our Schools

Gningue (2003) states in his abstract that "microcomputers and calculators are entering classrooms in substantial numbers, with attractive software applications, advanced multimedia capabilities and, above all, the World Wide Web. Research interest in the use of calculators and computing technologies in education has increased dramatically over the past decade because of this technology-enriched environment. In such settings, technology impacts not only what is taught and how it is taught but also what students learn and how they learn it. This has posed a challenge and an opportunity for teachers to learn more about computers and software, and how to integrate technology to teach or to enhance their curricula."

Gningue (2003) states, "Many students, because they play with the more complicated devices such as Nintendo 64 or Play Station 2, are more motivated to learn mathematics concepts using the TI-83 graphing calculator". A student in the study by Gningue, named Maria, goes even further by saying, "Students are so accustomed to playing with computer arcade games that the technology tools become like toys. The solution resides in an effective training of teachers toward the use of technology to enhance mathematics learning. …It is almost a sin not to make use of technology in the preparation of tomorrow's teachers and in the classroom. It is regrettable that many districts or schools do not provide graphing calculators, nor offer the training."

Forgasz (2006) and Benson et al. (2004) talk about reasons we aren't using the software to our biggest advantage right now in our schools, whether it be financial, lack of knowledge, or just not willing to change. Forgasz (2006), Benson et al. (2004), Gningue (2003) along with Okojie and Olinzock (2006) discuss ways to solve these problems with proper training and discuss short term and long term training needs.

Forgasz (2006) shows a table from a study by Smerdon, et al. (2000), that three of the most popular factors that encouraged teachers' use of computers for mathematics teaching are; Availability of computers or computer laboratories, Software (its quality, variety, motivation, fun and relevance), and Students enjoying using computers.

Benson, et al. (2004) says that "Teachers have a desire to provide the best education possible for their students". However, teachers do not want to fail, and thus, are not prone to trying new methods that they do not have confidence in.

Benson et al. (2004) of the impact of training in technology assisted instruction on skills and attitudes of pre-service teachers, found out that the training" makes a positive difference in the skills <u>and</u> attitudes of the pre-service teachers".

In a study by Gningue (2003), the researcher finds that while short term training is good at introducing new topics to teachers to use in their classroom, long term training is more practical because teachers are not only excited about something new but know how to use it more effectively.

Forgasz (2006) quotes facts from a study by Smerdon, et al. (2000), that the three most popular things preventing them from implementing technology were; Lack of release time to learn how to use computers or internet, Lack of time in daily schedule to use computers in class, and insufficient number of available computers.

Okojie and Olinzock (2006) try to find a solution by saying, "Developing positive mind-sets will assist teachers in achieving the spontaneity and the readiness necessary for technology integration. Okojie and Olinzock also tie in the training factor when they say, "It is a recognized fact that teachers need some kind of support as they implement technology in the classroom." Okojie and Olinzock go on further to say, "Teachers should be provided with the funds to carry out action research to improve the use of technology in their various classrooms and schools." Okojie and Olinzock also talk about a way to use in-service days when they say, "Schools should provide regular faculty development programs where experts in technology are invited to organize workshops and seminars to educate teachers on new technological products and their use." There are a lot of professional organizations that have sessions on using technology, such as graphing calculators and Geometer's Sketchpad. Okojie and Olinzock go further in suggesting school funding for these workshops by saying, "Teachers should be sponsored to attend professional conferences to participate in academic dialogues and assess their teaching methods and curriculum contents for relevance and currency in views of changes in technology".

The relationship between Dynamic Software and OELEs.

Hannafin (2004) reminds us that for almost 30 years, "many researchers have advocated using computer technology to create more learner-centered open-ended learning environments (OELEs) where learners are provided with varying amounts of help and support in deciding what they need to learn, how to learn, and what resources they need to learn it." Hannafin goes on to say that "proponents of OELEs argued that by identifying goals and constructive meanings, learners become active managers, rather than passive receptacles, of information."

We are then challenged with the quest of "What do open-ended learning environments provide learners with?" Hannafin later on in the citations answers this question. His response is that an OELE provides learners with opportunities to engage the environment in ways that support their unique needs and intentions for making sense of the world. Hannafin reminds us that OELEs do not provide predetermined content that follows structured learning objectives; rather they assist learners in defining goals and generating and revising appropriate learning strategies on the fly.

Before we get confused and start thinking that dynamic geometry software are open-ended learning environments, Hannafin reminds us that they are not. He very clearly explains the difference when he states "Dynamic Geometry programs are not OELEs in and of themselves, rather they are tools that can be used to create and support student-centered learning environments." Before any reader tries to draw inferences that are not consistent with Hannafin's study, the study Hannafin conducted used Geometer's Sketchpad.

Hannafin discusses why Geometer's Sketchpad is so useful later on by saying that "While these kinds of designer-orchestrated activities are perhaps more consistent with Rieber's description of an instructivist environment, some assert that students can learn to think critically and become better problem solvers, and are better able to draw and transfer meaning when afforded these opportunities." Hannafin (2004) concluded from his study that "Low-ability learners scored higher using Sketchpad than by traditional means".

CHAPTER 3: INFORMAL RESEARCH BASED UPON THE LITERATURE

Based on the research in Chapter 2, I decided to do some exploration and find further information about Fathom, TinkerPlots, and Cinderella.

Fathom

I will begin with Fathom. Research says that a unique feature about Fathom is that the data is held in a "collection", and is not necessarily represented by a table, as it is in most of the other programs. The collection holds the "gold balls" that are actually "cases", which are data from a specific person. Data can also be imported from a variety of sources, including spreadsheets and the internet, which allows more people to use Fathom with data that they already have. This way I can do a survey in my classroom with my students and have their data imported into Fathom to be analyzed. Working within the gold balls is also something that sets this software apart from others. The user works easily by clicking and dragging specific attributes to the correct axes and then graphing them. They are not limited as to what can be on each axis, which allows for a great deal of testing "what ifs?" by the user.

Grandgenett (2005) found that Fathom needs to be teacher-directed the first few times and its potential to enrich student learning. I can't agree more since it takes some time to get used to working within the program and using the "gold balls".

Fathom is also capable of providing simulations and getting into sophisticated analysis. Grandgenett (2005) was able in his study to simulate the national lottery. This is something that very few programs could ever do.

TinkerPlots

TinkerPlots was also a great program when dealing with data. TinkerPlots allows students to analyze data very easily. Each ball represents a case and if you click on a data you will see that case's data card. It allows you to quickly compare different data, such as gender, age, weight, etc… very quickly. The major advantage here is the speed with which one can compare a lot of data along different attributes". Having the ability to change icons is another plus to TinkerPlots. Another plus is the ability to import data from Microsoft Excel. I was able to try this out and it worked very easily. This is a great feature to anyone who does their spreadsheets using Excel. Students can begin by asking a question and then fill out the attributes on a Data Card. Students are also able to view more than one case at a time which is a huge plus when trying to make comparisons.

Cinderella

Cinderella, is a program from Germany that is able to help teach spherical geometry and hyperbolic geometry in a way that no other program can match.

With Cinderella, the user is able to explore the world of spherical geometry and still be able to relate his/her exploration findings to Euclidean geometry. The Lenart sphere allows a student to explore the basics of spherical geometry, but nothing allows a student to explore the "what ifs" of spherical geometry quicker than by using Cinderella. I have observed through personal experiences that Cinderella allows a person to truly see what a "line" is in Spherical

Geometry and explore other facets of spherical geometry with speed and ease. With Cinderella, the user is also able to explore the world of spherical geometry and still be able to relate his/her exploration findings to Euclidean geometry. Additionally, the user is able to explore the world of hyperbolic (saddle) geometry and relate the findings to Euclidean geometry. Cinderella also is able to work with complex conjugate which many other programs are not able to do.

Since I've already discussed the other forms of dynamic software, I will now talk about the main area of this paper, Geometer's Sketchpad. This is a more complete discussion about Geometer's Sketchpad and some activities I've done in class.

Geometer's Sketchpad

Every math conference I have attended, from state conferences to regional conferences to national conferences have had sessions dealing with Geometer's Sketchpad, but very rarely with Fathom, Cinderella, or TinkerPlots. Another advantage to Geometer's Sketchpad for me is that it is networkable.

Geometer's Sketchpad is not the final "know-it-all" of geometry; rather it is a tool to be used to further explore geometry at one's own pace and convenience. Like any new software to be used in education, Geometer's Sketchpad has its own tools, buttons, and menus which must be taught to the students via individual tutorials or full classroom presentations and follow-ups.

Manouchehri et al. (1998) talks about a progression through geometry at the pre-algebra level. They suggest starting with free exploration. The free exploration phase usually lasts about two or three days, and is the time when the students get familiar with the basics of Geometer's

Sketchpad. They are given handouts and work on their own, trying to see what will work and what won't work.

In my room, the use of Geometer's Sketchpad begins with an introductory phase, where the students are taught the basics of the tools, buttons and menus. Once students have learned the tools, buttons, and menus is when the real learning takes place.

The next phase is semi-structured activities. The semi-structured activities are tasks which are planned to facilitate students' discovery of specific geometric relationships or theorems. The first tasks done here are working with pre-constructed models, which the students make changes to and find new ideas. The second tasks involve the students construction of models with some instructions by the instructor. Again, the students try to find new ideas or prove old ones with their model. We are then able, for example, to prove that the sum of the angles of a triangle is 180 degrees, by using the animation features and the formula features. Teachers can use Geometer's Sketchpad to create learning environments where the students are able to help teach themselves by answering the "What if,,?" questions themselves with the help of Geometer's Sketchpad by using the animation and formula tools

I have used the animation and formula tools when we construct the nine-point circle in class and discuss what happens if we change some of the variables. I have also used Geometer's Sketchpad to teach all the different properties of triangles to students in a way they can understand using the animation and formula tools.

Finally, we have independent explorations and problem solving. These are more open-ended in the instructions. An example is "*Given a triangle, locate the midpoints of the sides.*

Connect the adjacent midpoints and determine if a relationship exists between the area of the triangle defined by the midpoints and the original triangle.".

I then researched other applications of Geometer's Sketchpad and found three that would be extremely difficult to do using traditional methods but were very easy to do with Geometer's Sketchpad.

Little (1999) has an activity that uses Geometer's Sketchpad at the Pre-Algebra level. Little's students are given six weeks to use Geometer's Sketchpad to create a Geometry Construction Manual using Geometer's Sketchpad. The instruction manual must contain the method of construction for each of the basic constructions, plus an enrichment item. Each page has a sequence of diagrams, in addition to written instructions. Grading is done based upon three components – Clarity of diagrams, Clarity of written instructions, and Overall organization and presentation. This is a good activity because it combines mathematics and technology in a way that forces the students to use technical writing to compose the instructions.

Dwyer et al. (1999) uses Geometer's Sketchpad to explore Hyperbolic Geometry by downloading the script tools at the web site theforum.swarthmore.edu/sketchpad/gsp.gallery/poincare/poincare.html. They show how their Poincare disk model is technically a Euclidean model of hyperbolic geometry. They then explore the following in Hyperbolic Geometry; the hyperbolic parallel postulate, a hyperbolic triangle, hyperbolic circles, circumcenter and the circumcircle, the centroid, orthocenter and incenter.

De Villiers (2005) describes an activity involving Viviani's theorem, which states that the sum of the distances from a point to the sides of an equilateral triangle is constant. The activity is; *A mathematical crocodile in the Okavango delta lives in a swampy region in the shape of an equi-angled pentagon. Since the crocodile captures prey an equal amount of times on each of the five banks, it wants to hide its captured prey where the sum of the five distances is the minimum. Where is this point?* DeVilliers even provides some help as a zipped Sketchpad sketch can be downloaded directly from: http://mysite.mweb.co.za/residents/profmd/crocodile.zip. At the end of De Villiers' explanation (proof) he shows a diagram which makes it easy to see. He also probes the readers further by asking it this is true in polyhedra also. De Villiers, from South Africa, goes even further by showing how it works in hyperbolic and elliptical geometry at the end of his article.

The low cost of Geometer's Sketchpad, the easy usability of Geometer's Sketchpad and the fact that we don't have to spend all of our time constructing things with pencil/compass but rather can explore what happens later after the constructions are made, all have made Geometer's Sketchpad popular with educators. Teachers in the past have been handcuffed because it took so much time to construct things and they were never able to adequately explore the "What if …" questions their students posed, except to answer them and have the students take their word for it - Geometer's Sketchpad allows the student to see it for themselves. This has emitted an exclamation from my students of "Now I finally understand why it works." After working with GSP, a teacher named Robert, in Gningue's (2003) study had the comment of "I finally learned something today."

Silver (1998) explores the question "Can computers be used to teach proofs?" and came up with the answer "yes and no" using Geometer's Sketchpad. In the traditional setting, they cannot, but in reality, they can. I have my students prove what the sum of a triangle is by constructing a triangle, measuring all the angles, and putting an equation on the screen adding all the angles so the user can see the sum. I then have the kids move a vertex so that they can see that the sum is still 180 degrees. I then have them use the animation keys to animate the part of the triangle so the students can still see the sum is 180. The low cost of Geometer's Sketchpad, the easy usability of Geometer's Sketchpad and the fact that we don't have to spend all of our time constructing things with pencil/compass but rather can explore what happens later after the constructions are made.

Dynamic Software

With all of this information and Hannafin (2004) telling us to use dynamic software in our classrooms, one would think that we all use it in our classrooms, but that is not the case. At International Falls High School, they only have one of the four types of software mentioned and it is used by only one of the two Geometry teachers. The Geometry teacher who does not use it said it is because "I am not comfortable with it." The Geometry teacher who does use it would like to use it further but needs to have more access to the computers than he currently has. In my experiences talking with other math teachers at summer classes, or at professional math conferences, teachers almost always fall into one of four categories; "I use it a lot", "We have it, but I don't feel comfortable enough to use it", "We have it but it's hard to get the computers", or "We can't afford it, but I'd love to have it".

CHAPTER 4: INTERPRETATION OF THE LITERATURE AND INFORMAL RESEARCH

The research that was collected, reviewed, and disseminated in this report show that changes need to be made to the curriculum used in our schools to improve student performance and that dynamic software is one method of change that has produced positive results.

Need for Technology

Manouchehri reminds us that a goal of mathematics instruction at the middle grades was to improve students so that mathematical thinking and their ability to communicate thoughts more clearly would improve, as stated by the National Council of Teachers of Mathematics already, in 1999.

We are also reminded that learning by doing has become the rule, rather than the exception in many classrooms. The research has shown us that we need to use a variety of tools to accomplish this goal. The traditional ways of lecturing for the class period and giving the assignment on the way out the door are changing to better meet the needs of the students. The changes in technology have been readily adapted by students in ways that make their lives better, so we, as instructors, need to change with the times and do what is best for the students, not what we are comfortable with. The students in their everyday lives are used to using "computers" in a number of different ways (i.e. graphing calculator, cell phone, camera, watch, desktop or laptop) from uploading or downloading pictures and graphics to playing games to chatting with their friends and we need to use their abilities and strengths in this area to increase their learning in our academic areas. Also, with the software being developed, students can explore many different types of probes into their subject areas, especially when all schools have computers.

The research has also shown that dynamic geometry software enables the students to further explore new topics and find new theorems or relationships. After the exploration phase, the students are able to explain their new "findings", which requires far more rigor than just answering a question or writing an essay. My own time as a tutor in the Mathematics/Computer Science Department at Bemidji State in 1986-1988 and subsequent 19 years of teaching have taught me that it is much harder to explain how to do a problem to someone else than to actually do the problem.

Need for Training

Since all the research, shows a need to use a variety of tools to help students learn and we also know from research that computers and dynamic software would be beneficial to the students and their grasping of the knowledge and working toward further explorations, there must be reasons why computers and dynamic software are not in every classroom. The research also tells teachers who do not want to fail and thus stick with what has worked for them in the past, rather than trying out new, untested ideas with their students.

Placing computers in a classroom does not always ensure their successful implementation. When I was hired in 1994, I was given a set of TI-81 calculators that had been purchased by the local Parent Teacher Association (PTA). However, the school I had been at the previous five years did not have graphing calculators and I did not know how to use them. Therefore, they were of no value to me until I learned how to use them. The research has talked about this dilemma when it discussed the three factors that were stopping teachers from using technology. The three factors were a lack of knowledge and time to learn, lack of time in the schedule to implement, and a lack of computers for everyone. Even though this research was initially done in 2000 and wrote about in 2006, those still seem to be the same three factors preventing implementation of technology in 2008 from discussions with colleagues.

All three seem to flow together if the first factor is accomplished. A teacher who feels they have a strong base of knowledge will probably find the time in the schedule to implement and then talk to administration and community members about the need for more computers.

Training would help the teachers accomplish that first goal and school districts have a number of ways at their disposal in order to accomplish this. The research shows us the need for training because it makes a positive difference in skills and attitudes of the staff. We were also reminded that if a positive mind-set is developed, then the teachers are more willing to integrate technology into their lesson plans. When training, the research reminded us that short term training is good for introducing something, but long term training will promote a more effective use of the technology being learned.

One way of training mentioned in the research was to use in-service days for this use. International Falls School District has done this in the past with some very strong positive results. This allows the district/school to bring in the trainer for the technology/software that will benefit them the most and to get the hands-on training that is so beneficial, since it was already mentioned that we learn best by doing rather than by listening.

Another method mentioned for training was to provide the teacher with the funds necessary for purchasing software/computers and also allow the teachers to attend professional conferences, where the teacher can learn new technologies and be exposed to ways of learning they have not seen before so they can return a better teacher for the district.

Participating in academic dialogues is also a method of learning new ideas or improving on what a person knows. Allowing the teacher the time and money to go to a professional conference will open up a lot of doors and give them the opportunity to meet other teachers and allow the teacher to have strong, positive, learning dialogues.

Geometer's Sketchpad

Geometer's Sketchpad is one of the most popular of the types of dynamic software for the students to learn with. Geometer's Sketchpad was shown during both the review of the literature and the informal research based on the literature to be beneficial in all situations. The research showed that Geometer's Sketchpad makes learning more available to all students and even gave an example of "Amanda" and how Geometer's Sketchpad accommodated her need for more time to study the information being presented and for more time to process the information. This is a huge benefit in these days of "No Child Left Behind" and trying to make learning the best for all students in classrooms that are integrated with students of all levels, including sight-impaired students, hearing-impaired students, students with different types of other learning disabilities, or gifted and talented students all in the same area. The research also showed that pairs using Geometer's Sketchpad did better than pairs using traditional means. Finally, the research was able to show different ways that Geometer's Sketchpad was able to do problems that traditional means either could not do, or could not do and allow for the student to do further investigation with measured success.

Other types of dynamic software

The research showed three other main types of dynamic software. These three are Fathom, TinkerPlots, and Cinderella.

Fathom can be a very useful tool in the classroom for students to learn data concepts and probe deeper into critical thinking skills Fathom allows the user to import data from other sources, such as spreadsheets and the internet or to use data that is already made up or is made up by the user. Being able to export real-life data from a spreadsheet allows the user to make inferences and conclusions using real-life data rather than made up numbers. The data is held in a collection, consisting of gold balls, which allows the user to easily maneuver within different cases. The users are able to "pick and choose" among the attributes and drag the ones they want to compare to the correct axes for graphing and analysis. The ease and simplicity to pick the attributes and drag them to the axes for comparison allows the user to make a lot of comparisons in a short amount of time. The ease and time factors allow the user to make more comparisons of different attributes and thereby, arrive at more complete conclusions. Due to this, more factors will be able to be eliminated during the testing of hypotheses, resulting in stronger conclusions. Another use for a class is to have the students independently use the same data to arrive at conclusions and then have the class discuss their conclusions to see how different people interpreted the same data. Finally, Grandgenett was able to simulate the National Lottery, which shows that Fathom can do what many other programs cannot. While being teacher-directed at first to get used to the program, the students later will be allowed to take a set of data they have collected and come up with their own conclusions.

TinkerPlots is also a very useful program when analyzing data. TinkerPlots uses a data card so that one could see all the data from a person at the same time. This can be accomplished by having the students fill out a questionnaire about data (height, weight, age, sex, shoe size, etc…) and then have the class results made into a spreadsheet using Mircrosoft Excel. Then the students can export their Microsoft Excel data into TinkerPlots and make comparisons by testing a variety of hypotheses. This allows the students to be self-driven and for the activity to be open-ended because the teacher can have the students find conclusions on their own. The ability to change icons using TinkerPlots allows the user to set things up how the user wants to for better maneuverability within the data. TinkerPlots allows the students to advance to a level of problem solving that could not have been done before with such ease. Finally, TinkerPlots has the ability to see different cases at the same time, which allow the user to reflect back upon past observations and continue to make more hypotheses and test new ideas. Like Fathom, TinkerPlots will need to be teacher-directed the first few times but after that the ease of the program will allow the users to explore quickly on their own.

Cinderella is a program that allows students to be able to see in a third dimension and with ease that is difficult to normally envision. Cinderella allows the user to make figures in either Euclidean Geometry, Spherical Geometry, or Hyperbolic Geometry and then compare how they look in one of the other two with ease. While other objects, such as the Lenart Sphere or a ball, can help with Spherical Geometry, or a saddle can help with Hyperbolic (Saddle) Geometry, they do not have the quick capabilities for comparison and the ease to make changes that Cinderella offers the user. The measurement tools also take out the "human error" part of a task that limits the potential testing of hypotheses and allows the user to test all hypotheses and draw more accurate conclusions. Cinderella's biggest advantage is the ease at which a user is able to draw a figure and then see how it looks in the other two types of Geometry. This is something that sets Cinderella apart from other programs.

CHAPTER 5: CONCLUSIONS

Can Dynamic Software Improve Student Learning?

The research on this topic seemed to be quite clear. Gningue (2003) reminded us that we need to use technology because students are used to it and Yelland (1999) talked about how learning by doing is now the rule, whereby it used to be the exception. Their research shows that we need to use not only technology, but more importantly, dynamic software, to bring out the best in the students. Edwards (Spring 2005) also agreed when he said that dynamic geometry software enables us in that the questions are as important as answers and in which explaining one's ideas rigorously is as important as writing two-column proofs. Finally, Santos-Trigo (2004) said that with dynamic software, students are able to discover new theorems or relationships and are able to engage in a way of thinking that goes beyond reaching a particular solution or response to a particular problem. So, we have learned that dynamic software can improve student learning because it allows us to learn by doing and to be able to establish new theorems or relationships.

Can Geometer's Sketchpad promote learning with different levels of learners?

Shaw, Durden and Baker (1998), along with Hannafin (2004) researched and found that Geometer's Sketchpad made learning more available for all students. They also showed an example where Geometer's Sketchpad allowed a student the needed time to study and the needed time to process the information. This is possible because with Geometer's Sketchpad a student can be given an assignment and then can work at their own pace. The slower learners are allowed to work at a slower pace, the medium learners are allowed to work at a medium pace and the fast learners are allowed to work at their own pace. I have given an assignment and had students work on it in resource room with their resource teacher, in my room before or after school, or just in class. Geometer's Sketchpad allows a student to save the project where they are at, and then resume it at a later time. It also allows the faster paced student to probe further into the situation with bothering the slower paced student that is not that far in their work. Geometer's Sketchpad promotes learning with different levels of learners.

Does Geometer's Sketchpad help students probe deeper into questions and further critical thinking?

The research shows that Geometer's Sketchpad allows one to probe "What if…" questions more effectively. Yelland (1999) and Hannafin (2004) continue to show that critical thinking is aided by Geometer's Sketchpad. Little (1999) shows a technical writing assignment and Manouchehri et al. (1998) showed us ways that Geometer's Sketchpad can be used to further probe questions and further critical thinking skills. Geometer's Sketchpad allows varied multi-response questions, such as "Show how much the sum of the angles of a triangle are and any other things you can find out about a triangle?" Dwyer et al. (1999) uses Geometer's Sketchpad to explore Hyperbolic Geometry and De Villiers (2005) uses Geometer's Sketchpad to probe into a very difficult question. This research showing the many different ways to promote critical thinking shows us that Geometer's Sketchpad helps students probe deeper into questions and further critical thinking skills.

Can Geometer's Sketchpad be used as an effective tool to help students learn more?

For a tool to be effective, it should promote learning across all levels and probe deeper into questions and allow the user to further critical thinking skills.

Dynamic Software has shown many times that it improves learning and that Geometer's Sketchpad, a type of dynamic software, promotes learning with different levels of learners. This was shown through the study of Amanda, the cerebral palsy student through the applicability at the junior high and elementary levels and finally, the applicability of Geometer's Sketchpad with classes other than Geometry, such as Algebra 1, Algebra 2, and Calculus.

Also, it has been shown that Geometer's Sketchpad helps students probe deeper into questions and further critical thinking skills. It allows the user to answer an open-ended question of "Show what the sum of the angles of a triangle is", rather than just answering the traditional recall question of "What is the sum of the angles of a triangle". Geometer's Sketchpad allows the user to find out things for themselves rather than relying on other people to tell them everything. It allows the user to explore the relationships between the Orthocenter, Incenter and Circumcenter of a triangle, rather than being told everything.

Therefore, the research shows that Geometer's Sketchpad can be used as an effective tool to help students learn more since it promotes learning across all levels and allows the user to probe deeper into questions and to further critical thinking skills.

What I will do next because of the research

Following the end of my research paper, I plan on implementing more Geometer's Sketchpad activities into my classroom learning. I also plan on comparing what I do with what Manouchehri et al. (1998) says about the exploration phases (Free, Semi-Structured, Independent Exploration with Problem Solving) and trying to find where I need to change my curriculum to meet the needs of all three phases.

I have already incorporated more assignments with my seventh grade classes with Geometer's Sketchpad and will continue to use it to enhance their learning. I will also use it with my Algebra 2 classes using some activities I found during my research. I will also be sharing the activities I have found with my colleagues so they can see what would work for them.

I will also be lobbying the school district for more computers as we have the same problem most schools have – a lack of computers to go around. We have two labs for 40 teachers and that is not enough with the technology, more and more, classes are using.

If all is successful, I will present some activities at a Minnesota Council of Teachers of Mathematics State Conference.

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